

# Calculus

15 / 10 / 2022

$$* y = f(x)$$

$$* y' = \frac{dy}{dx} = D_y = f'(x) = Df(x)$$

$$1 \frac{d}{dx} [c] = 0$$

$$\text{ex. } y = 5 \Rightarrow y' = 0$$

$$2 \frac{d}{dx} [x] = 1$$

$$3 \frac{d}{dx} [x^n] = n x^{n-1}$$

$$\text{ex. } y = 5x^3 + 3x^2 + 4x + 2$$

$$y' = 15x^2 + 6x + 4$$

$$4 \frac{d}{dx} [\sqrt{x}] = \frac{1}{2\sqrt{x}}$$

$$\text{ex. } y = 3x^{-2} + 3\sqrt{x} + 13$$

$$y' = -6x^{-3} + \frac{3}{2\sqrt{x}}$$

$$\text{ex. } y = x^8 - 2x^5 + 6x + 1$$

$$y' = 8x^7 - 10x^4 + 6$$

$$y'' = 56x^6 - 40x^3$$

$$y''' = 336x^5 - 120x^2$$

$$y^{(4)} = 1680x^4 - 240x$$

Good!

$$5 \quad y = f \cdot g$$

$$y' = f \cdot g' + f' \cdot g$$

$$\text{ex. } y = (4x^2 - 1)(7x^3 + x)$$

$$y' = 8x(7x^3 + x) + (4x^2 - 1)(21x^2 + 1)$$

$$6 \quad y = \frac{f}{g}$$

$$y' = \frac{f' \cdot g - g' \cdot f}{g^2}$$

$$\text{ex. } y = \frac{x^3 + 2x^2 - 1}{x + 5}$$

$$y' = \frac{(3x^2 + 4x)(x + 5) - (x^3 + 2x^2 - 1)}{(x + 5)^2}$$

$$7 \quad y = (f(x))^n$$

$$y' = n(f(x))^{n-1} \cdot f'(x)$$

$$\text{ex. } y = (x^4 + 3x^2 + 5)^{10}$$

$$Dy = 10(x^4 + 3x^2 + 5)^9 \cdot 4x^3 + 6x$$

$$8 \quad y = \sqrt{f(x)}$$

$$y' = \frac{1}{2\sqrt{f(x)}} \cdot f'(x)$$

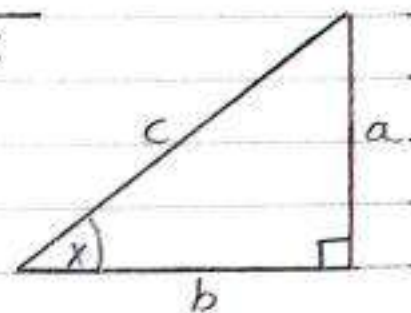
$$\text{ex. } y = \sqrt{x^2 + 1}$$

$$y' = \frac{1}{2\sqrt{x^2 + 1}} \cdot 2x$$

$$\sin(x) = \frac{a}{c} \longrightarrow \operatorname{cosec}(x) = \frac{1}{\sin x}$$

$$\cos(x) = \frac{b}{c} \longrightarrow \sec(x) = \frac{1}{\cos x}$$

$$\tan(x) = \frac{a}{b} \longrightarrow \cot(x) = \frac{1}{\tan x}$$



| Function                 | Differentiation                        |
|--------------------------|----------------------------------------|
| $\sin x$                 | $\cos x$                               |
| $\cos x$                 | $-\sin x$                              |
| $\tan x$                 | $\sec^2 x$                             |
| $\sec x$                 | $\sec x \cdot \tan x$                  |
| $\operatorname{cosec} x$ | $-\operatorname{cosec} x \cdot \cot x$ |
| $\cot x$                 | $-\operatorname{cosec}^2 x$            |

ex.  $y = x \sin x$

$$y' = 1 \cdot \sin x + x \cdot \cos x$$

ex.  $y = \frac{\sin x}{1 + \cos x}$

$$y' = \frac{\cos x(1 + \cos x) - (\sin x)(-\sin x)}{(1 + \cos x)^2} = \frac{\cos x + \cos^2 x + \sin^2 x}{(1 + \cos x)^2}$$

$$= \frac{1 + \cos x}{(1 + \cos x)^2} = \frac{1}{1 + \cos x}$$

ex.  $y = \sqrt{1+x^3 + \tan x}$

$$y' = \frac{1}{2\sqrt{1+x^3 + \tan x}} \cdot (3x^2 + \sec^2 x)$$

[9]  $\frac{d}{dx} [f(g(x))] = f'(g(x)) \cdot g'(x)$

ex.  $y = \cos(x^3)$

$$y' = -\sin(x^3) \cdot (3x^2)$$

ex.  $w = \tan(4t^3 + t)$

$$\frac{dw}{dt} = \sec^2(4t^3 + t) \cdot (12t^2 + 1)$$

ex.  $y = \sin \sqrt{\tan(x^2+1)}$

$$y' = \cos \sqrt{\tan(x^2+1)} \cdot \frac{1}{2\sqrt{\tan(x^2+1)}} \cdot \sec^2(2x)$$

## Implicit Differentiation

تفاضل ضمني

$$y = f(x)$$

$$y = x^2 + 1$$

$$\rightarrow y' = 2x$$

دالة صريحة

$$y = \sin x + \sqrt{x} + 1 \rightarrow y' = \cos x + \frac{1}{2\sqrt{x}}$$

$$x^2 + y^2 + xy = 1 \rightarrow \text{دالة ضمنية}$$

$$2x + 2y \cdot y' + y + xy' = 0$$

$$(2x + y) + y'(2y + x) = 0$$

$$y'(2y + x) = -(2x + y)$$

$$y' = \frac{-(2x + y)}{2y + x}$$

$$\text{ex. } 5y^2 + \sin y = x^2$$

$$10y \cdot y' + y' \cdot \cos y = 2x$$

$$y'(10y + \cos y) = 2x$$

$$y' = \frac{2x}{10y + \cos y}$$

$$\text{ex. } 4x^2 + 2y^3 + xy = 5$$

$$8x + 6y^2 \cdot y' + y + xy' = 0$$

$$y'(6y^2 + x) = -8x - y$$

$$y' = \frac{-8x - y}{6y^2 + x}$$

$$\star y = n^x \implies y' = n^x \cdot \ln n$$

$$\text{ex. } y = 3^x \implies y' = 3^x \cdot \ln 3$$

$$\star y = n^{f(x)} \longrightarrow y' = n^{f(x)} \cdot f'(x) \cdot \ln n$$

$$\text{ex. } y = 5^{\sqrt{x}}$$

$$y' = 5^{\sqrt{x}} \cdot \frac{1}{2\sqrt{x}} \cdot \ln 5$$

$$\text{ex. } y = 2^{\tan x} \longrightarrow y' = 2^{\tan x} \cdot \sec^2 x \cdot \ln 2$$

$$\star y = e^x \longrightarrow y' = e^x$$

$$\star y = e^{f(x)} \longrightarrow y' = e^{f(x)} \cdot f'(x)$$

$$\text{ex. } y = e^{-x} \longrightarrow y' = -e^{-x}$$

$$y = e^{\sin x} \longrightarrow y' = e^{\sin x} \cdot \cos x$$

$$y = x e^{\tan x}$$

$$y' = e^{\tan x} + x e^{\tan x} \cdot \sec^2 x$$

Ln خواص

"مهم"

$$*y = \ln x$$

$$[1] \ln ab = \ln a + \ln b$$

$$[2] \ln \frac{a}{b} = \ln a - \ln b$$

$$[3] \ln a^n = n \ln a$$

$$[4] \ln e = 1$$

$$[5] \ln e^x = x$$

$$[6] e^{\ln x} = x$$

$$[7] \ln 1 = 0$$

$$[8] \ln \frac{1}{a} = \ln a$$

$$[9] y = \ln x$$

$$y' = \frac{1}{x}$$

$$[10] y = \ln f(x)$$

$$y' = \frac{f'(x)}{f(x)}$$

ex.  $y = \ln \sin x$

$$y' = \frac{\cos x}{\sin x} = \cot x$$

ex.  $y = \ln (x^3 + 3x + 1)^4 = 4 \ln (x^3 + 3x + 1)$

$$y' = 4 \frac{3x^2 + 3}{x^3 + 3x + 1}$$

Find  $\frac{d}{dx} [\ln (x^2 + 1)]$

$$\frac{d}{dx} = \frac{2x}{x^2 + 1}$$

ex.  $y = \ln \left( \frac{x^2 \sin x}{\sqrt{1+x}} \right)$

$$= \ln x^2 \sin x - \ln \sqrt{1+x}$$

$$= \ln x^2 + \ln \sin x - \frac{1}{2} \ln (1+x)$$

$$y' = \frac{2}{x} + \frac{\cos x}{\sin x} - \frac{1}{2} \cdot \frac{1}{1+x}$$

ex.  $y = \ln [\sin (3^{\sqrt{\tan x}})]$

$$y' = \frac{1}{\sin (3^{\sqrt{\tan x}})} \cdot \cos 3^{\sqrt{\tan x}} \cdot 3^{\sqrt{\tan x}} \cdot \ln 3 \cdot \frac{1}{2\sqrt{\tan x}} \cdot \sec^2 x$$

$$① y = e^{f(x)} \Rightarrow y' = e^{f(x)} \cdot f'(x)$$

$$② y = a^{f(x)} \Rightarrow y' = a^{f(x)} \cdot f'(x) \cdot \ln a$$

$$③ y = (f(x))^{g(x)}$$

أخذنا للطرفين

$$\ln y = \ln (f(x))^{g(x)}$$

$$\ln y = g(x) \ln f(x)$$

$$\frac{y'}{y} = g(x) \cdot \frac{1}{f(x)} \cdot f'(x) + \ln f(x) \cdot g'(x)$$

$$y' = y \left( g(x) \cdot \frac{f'(x)}{f(x)} + \ln f(x) \cdot g'(x) \right)$$

$$\therefore y' = g(x) \cdot \frac{f'(x)}{f(x)} + \ln f(x) \cdot g'(x) \cdot (f(x))^{g(x)}$$

$$\text{ex. } y = x^x$$

أخذنا للطرفين

$$\ln y = \ln(x^x)$$

$$\ln y = x \ln x$$

$$\frac{y'}{y} = x \cdot \frac{1}{x} + \ln(x)$$

$$y' = (1 + \ln(x)) \cdot x^x$$

ex.  $y = (x^2 + 1)^{\sin(x)}$

$$\ln y = \ln (x^2 + 1)^{\sin(x)}$$

$$\ln y = \sin x \ln (x^2 + 1)$$

$$\frac{y'}{y} = \sin x \cdot \frac{2x}{x^2 + 1} + \ln (x^2 + 1) \cos x$$

$$y' = \left( \sin x \cdot \frac{2x}{x^2 + 1} + \ln (x^2 + 1) \cos x \right) \cdot y$$

$$y' = \left( \sin x \cdot \frac{2x}{x^2 + 1} + \ln (x^2 + 1) \cos(x) \right) \cdot (x^2 + 1)^{\sin x}$$

ex.  $y = \ln(x)^{\sin^2(x)}$

$$y = \sin^2(x) \ln(x)$$

$$y' = \sin^2(x) \cdot \frac{1}{x} + \ln(x) \cdot 2 \sin(x) \cdot \cos(x)$$

"معادلة خط التماس"

$$y - y_0 = m(x - x_0)$$

$y_0$  هي قيمة الدالة عند  $x_0$

$$m \Big|_{(x_0, y_0)} = y' \Big|_{(x_0, y_0)}$$

Ex.  $y = \ln(x^2 + 1) \quad x_0 = 1$

$$y_0 = \ln(1^2 + 1) = \ln(2)$$

$$y' = \frac{2x}{x^2 + 1}$$

$$m = y' \Big|_{(1, \ln 2)} = \frac{2 \cdot 1}{1^2 + 1} = 1$$

$$y - \ln(2) = x - 1 \Rightarrow y - x = \ln(2) - 1 \quad \text{معادلة}$$

Ex.  $y^3 + 2yy^2 + xy + 1 = 0 \quad \text{point } (1, 1)$

$$y - y_0 = m(x - x_0)$$

$$3y^2 \cdot y' + 2(y \cdot 2y \cdot y' + y^2 \cdot y') + xy' + y = 0$$

$$y' [3y^2 + 2y^2 + 2y^2 + x] + y = 0$$

$$y' = \frac{-y}{3y^2 + x}$$

## الدوال العكسية

$$\star \sin^{-1}(x) \neq (\sin x)^{-1} \neq \sin x^{-1}$$

دالة عكسية

$$y' = -(\sin x)^{-2} \cdot \cos x$$

$$y' = \cos(x)^{-1} \cdot -x^{-2}$$

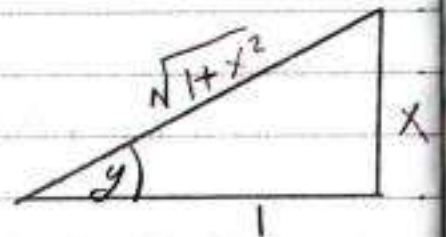
$$\boxed{1} \tan^{-1}(x) \Rightarrow y' = \frac{1}{1+x^2} \quad \text{مشتقة الزاوية}$$

$$\tan^{-1}(x) \Rightarrow y' = \frac{1}{1+x^2}$$

$$y = \tan^{-1}(x)$$

$$\tan y = \tan \tan^{-1}(x)$$

$$\tan y = x$$



$$y' \cdot \sec^2 y = 1 \Rightarrow y' = \frac{1}{\sec^2 y} = \cos^2 y$$

$$\cos^2 y = y' = \left( \frac{1}{\sqrt{1+x^2}} \right)^2 = \frac{1}{1+x^2}$$

$$\boxed{2} y = \sec^{-1}(x) \quad y' = \frac{1}{\sqrt{(زاوية)^2 - 1}} = \frac{1}{x \sqrt{x^2 - 1}}$$

$$y = \sec^{-1} x$$

$$\sec y = \sec \sec^{-1} x$$

$$\sec y = x$$

$$\sec y \tan y \cdot y' = 1 \quad y' = \frac{1}{\sec y \tan y} = \frac{1}{x \sqrt{x^2 - 1}}$$

الزاوية

| الدالة         | التفاضل                   | الدالة                         | التفاضل                    | ملاحظات |
|----------------|---------------------------|--------------------------------|----------------------------|---------|
| $\sin^{-1}(x)$ | $\frac{1}{\sqrt{1-x^2}}$  | $\cos^{-1}(x)$                 | $\frac{-1}{\sqrt{1-x^2}}$  |         |
| $\sec^{-1}(x)$ | $\frac{1}{x\sqrt{x^2-1}}$ | $\operatorname{cosec}^{-1}(x)$ | $\frac{-1}{x\sqrt{x^2-1}}$ |         |
| $\tan^{-1}(x)$ | $\frac{1}{1+x^2}$         | $\cot^{-1}(x)$                 | $\frac{-1}{1+x^2}$         |         |

Ex.  $y = \sec^{-1} e^x$

$$y' = \frac{1}{e^x \sqrt{e^{2x}-1}} \cdot e^x = \frac{1}{\sqrt{e^{2x}-1}}$$

Ex.  $y = \tan^{-1}(\ln(x) + \sin x^2)$

$$y' = \frac{1}{1+(\ln(x) + \sin x^2)^2} \cdot \frac{1}{x} + \cos x^2 \cdot 2x$$

Ex.  $y = e^{\operatorname{cosec}^{-1} \sqrt[3]{x}}$

$$y' = e^{\operatorname{cosec}^{-1} \sqrt[3]{x}} \cdot \frac{-1}{3\sqrt[3]{x} \sqrt{(x^{\frac{1}{3}})^2-1}} \cdot \frac{1}{3} x^{-\frac{2}{3}}$$

$$y' = \frac{-e^{\operatorname{cosec}^{-1} \sqrt[3]{x}}}{3\sqrt[3]{x} \sqrt{(x^{\frac{1}{3}})^2-1}} \cdot \frac{1}{3} x^{-\frac{2}{3}}$$

#

# Calculus

## Lecture 4

تطبيقات على التفاضل

5.11.2022

$$\text{Ex. } f(x) = x^3 - 3x + 1$$

أوجد النقاط الحرجة للدالة  
"critical point"

$$f'(x) = 3x^2 - 3$$

$$f'(x) = 0 \Rightarrow 3x^2 - 3 = 0 \Rightarrow x^2 = 1 \Rightarrow x = \pm 1$$

$$\text{Ex. } f(x) = 3x^{\frac{5}{3}} - 15x^{\frac{2}{3}}$$

$$f'(x) = 5x^{\frac{2}{3}} - 10x^{-\frac{1}{3}}$$

$$= 5x^{-\frac{1}{3}}(x-2)$$

$$f'(x) = 0 \Rightarrow 5x^{-\frac{1}{3}}(x-2) = 0$$

$$x^{-\frac{1}{3}} \mid x-2=0$$

$$x=0 \mid x=2$$

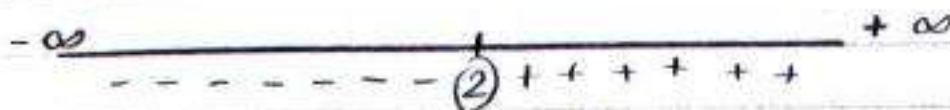
## 2. Increasing & Decreasing intervals فترات التزايد والتناقص

$$f(x) = x^2 - 4x + 3$$

التعويض في المشتقة  
وليسه الدالة

$$f'(x) = 2x - 4$$

$$f'(x) = 0 \Rightarrow 2x - 4 = 0 \Rightarrow \boxed{x = 2}$$



$$f'(x) < 0$$

$$(-\infty, 2]$$

decreasing interval

$$f'(x) > 0$$

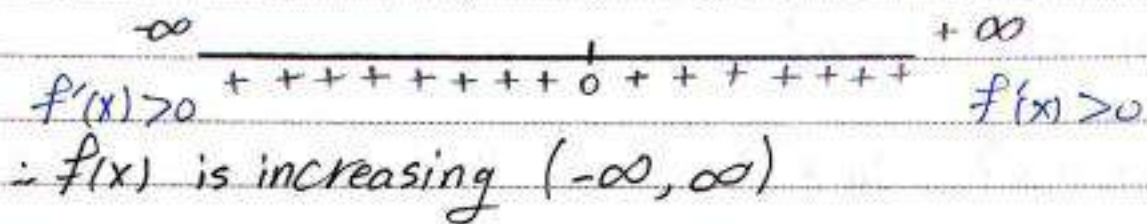
$$[2, \infty)$$

increasing interval

Ex.  $f(x) = x^3$

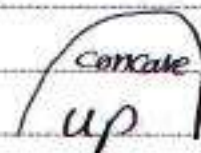
$f'(x) = 3x^2$

$f'(x) = 0 \Rightarrow 3x^2 = 0 \Rightarrow x = 0 \leftarrow \text{critical point}$



### 3 Concavity:

$f(x) = x^3$

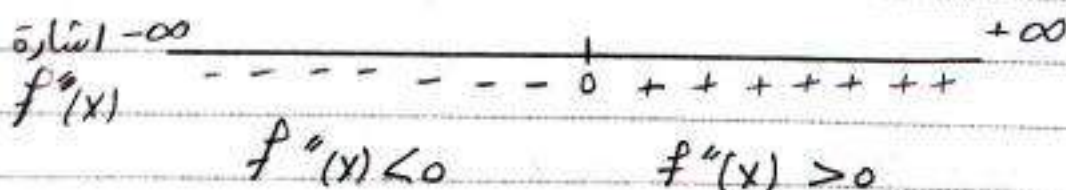


$f'(x) = 3x^2$

التحويل في المشتقة الثانية \*

$f''(x) = 6x$

$f'(x) = 0 \Rightarrow 3x^2 = 0 \Rightarrow x = 0 \leftarrow \text{critical point}$



$(-\infty, 0]$   
concave down

$[0, \infty)$   
concave up



$$f(x) = x^4 - 2x$$

$$f'(x) = 4x^3 - 2$$

$$f''(x) = 12x^2$$

$$f''(x) = 0 \Rightarrow x = 0$$



$$f''(x) > 0$$

$$f''(x) > 0$$

no inflection point

maxima, minima

extrema

$$f(x) = 3x^5 - 5x^3$$

$$f'(x) = 15x^4 - 15x^2$$

$$f'(x) = 0 \Rightarrow 15x^4 - 15x^2 = 0 \Rightarrow$$

$$\begin{aligned} x &= -1 \\ x &= 0 \\ x &= 1 \end{aligned}$$



maximum  
Value

minimum  
Value

$$x = 1 \Rightarrow \text{maximum}$$

$$x = 0 \Rightarrow \text{no extrema}$$

$$x = -1 \Rightarrow \text{minimum}$$

Ex.  $f(x) = 2x^3 - 15x^2 + 36x$  on intervals

$$f'(x) = 6x^2 - 30x + 36$$

$$f'(x) = 0 \quad 6x^2 - 30x + 36 = 0 \quad (\div 6)$$

$$x^2 - 5x + 6 = 0$$

$$(x-2)(x-3) = 0$$

$$x = 2 \mid x = 3 \quad \text{critical point}$$

absolute maximum value

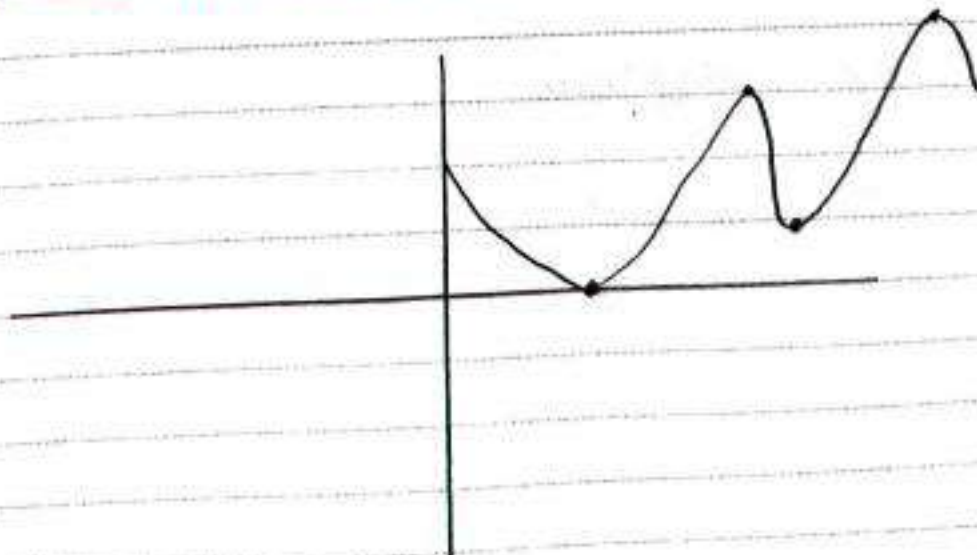
absolute minimum value

$$f(1) = 23 \rightarrow \text{absolute minimum}$$

$$f(2) = 28$$

$$f(3) = 27$$

$$f(5) = 55 \rightarrow \text{absolute maximum}$$

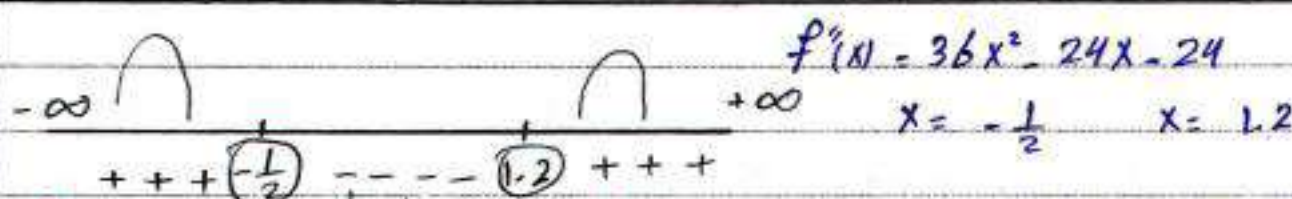
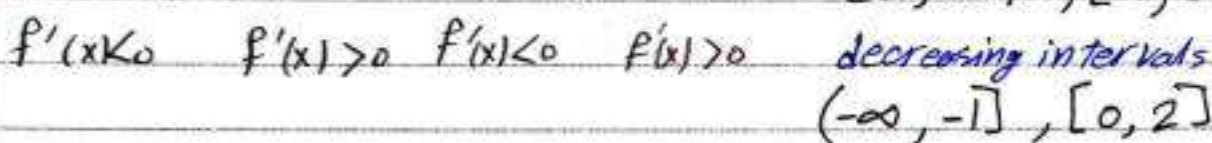
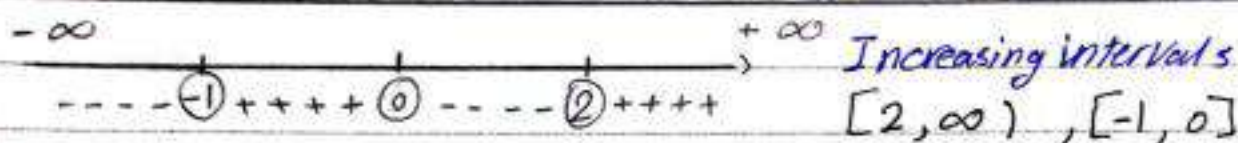


ex.  $f(x) = 3x^4 - 4x^3 - 12x^2 + 5$

$f'(x) = 12x^3 - 12x^2 - 24x \Rightarrow f'(x) = 0$  (1)

critical points

$\Rightarrow 12x(x+1)(x-2) = 0 \quad x = 0, x = -1, x = 2$



concave down  $(-\infty, -\frac{1}{2}], [1.2, \infty)$   
 concave up  $[-\frac{1}{2}, 1.2]$

Inflection points  $x = -\frac{1}{2}, x = 1.2$  (4)

$f(0) = 5$  (5)

$f(-1) = 6$  maximum

$f(2) = -27$  minimum

\*السؤال الثالث: الزاوية.

$$\sinh x = \frac{e^x - e^{-x}}{2}$$

$$\operatorname{cosech} x = \frac{1}{\sinh x} = \frac{2}{e^x - e^{-x}}$$

$$\cosh x = \frac{e^x + e^{-x}}{2}$$

$$\operatorname{sech} x = \frac{1}{\cosh x} = \frac{2}{e^x + e^{-x}}$$

$$\tanh x = \frac{\sinh x}{\cosh x} = \frac{e^x - e^{-x}}{e^x + e^{-x}}$$

$$\operatorname{coth} x = \frac{1}{\tanh x} = \frac{e^x + e^{-x}}{e^x - e^{-x}}$$

$$\begin{aligned} \text{Ex. } \sinh x + \cosh x &= \frac{e^x - e^{-x}}{2} + \frac{e^x + e^{-x}}{2} = \frac{e^x - e^{-x} + e^x + e^{-x}}{2} \\ &= \frac{2e^x}{2} = e^x \end{aligned}$$

\*important Notes:

$$① \sinh x + \cosh x = e^x$$

$$⑧ \sinh(0) = \frac{e^0 - e^{-0}}{2} = 0$$

$$② \sinh x - \cosh x = e^{-x}$$

$$⑨ \cosh(0) = \frac{e^0 + e^{-0}}{2} = \frac{2}{2} = 1$$

$$③ \cosh^2 x - \sinh^2 x = 1$$

$$⑩ \tanh(0) = 0$$

$$④ 1 - \tanh^2 x = \operatorname{sech}^2 x$$

$$⑤ \operatorname{coth}^2 x - 1 = \operatorname{cosech}^2 x$$

$$⑥ \cosh(-x) = \cosh(x)$$

$$⑦ \sinh(-x) = -\sinh x$$

تضاف هذه الدوال القبلية الزائدة

| $f(x)$                    | $f'(x)$                                  |
|---------------------------|------------------------------------------|
| $\sinh x$                 | $\cosh x$                                |
| $\cosh x$                 | $\sinh x$                                |
| $\tanh x$                 | $\operatorname{sech}^2 x$                |
| $\operatorname{sech} x$   | $-\operatorname{sech} x \cdot \tanh x$   |
| $\operatorname{cosech} x$ | $-\operatorname{cosech} x \cdot \coth x$ |
| $\coth x$                 | $-\operatorname{cosech}^2 x$             |

Ex.  $y = \ln(\tanh \sqrt{x^2+1})$

$$y' = \frac{\operatorname{sech}^2 \sqrt{x^2+1}}{\tanh \sqrt{x^2+1}} \cdot \frac{1}{2\sqrt{x^2+1}} \cdot 2x$$

$$\therefore y' = \frac{x \operatorname{sech}^2 \sqrt{x^2+1}}{\sqrt{x^2+1} \tanh \sqrt{x^2+1}}$$

Ex.  $y = \sqrt{\coth(\ln \sqrt{x} + x^2)}$

$$y' = \frac{-\operatorname{cosech}^2(\ln \sqrt{x} + x^2)}{2\sqrt{\coth(\ln \sqrt{x} + x^2)}} \cdot \left( \frac{1}{\sqrt{x}} \cdot \frac{1}{2\sqrt{x}} + 2x \right)$$

$$y' = \frac{-\operatorname{cosech}^2(\ln \sqrt{x} + x^2)}{4\sqrt{\coth(\ln \sqrt{x} + x^2)}} + 2x$$

Ex.  $y = 5^{\tanh x}$

$$y' = 5^{\tanh x} \cdot \operatorname{sech}^2 x \cdot \ln 5$$

"الموال الزائدة العكسية"

$$① \sinh^{-1} x = \ln(x + \sqrt{x^2 + 1})$$

$$② \cosh^{-1} x = \ln(x + \sqrt{x^2 - 1})$$

$$③ \tanh^{-1} x = \frac{1}{2} \ln\left(\frac{1+x}{1-x}\right)$$

$$④ \coth^{-1} x = \frac{1}{2} \ln\left(\frac{1+x}{x-1}\right)$$

$$⑤ \operatorname{sech}^{-1} x = \ln\left(\frac{1 + \sqrt{1-x^2}}{x}\right)$$

$$⑥ \operatorname{cosech}^{-1} x = \ln\left(\frac{1 + \sqrt{1+x^2}}{x}\right)$$

| $f(x)$                       | $f'(x)$                    | $f(x)$                         | $f'(x)$                    |
|------------------------------|----------------------------|--------------------------------|----------------------------|
| $\sinh^{-1} x$               | $\frac{1}{\sqrt{1+x^2}}$   | $\cosh^{-1} x$                 | $\frac{1}{\sqrt{x^2-1}}$   |
| $\tanh^{-1} x$               | $\frac{1}{1-x^2}$          | $\coth^{-1} x$                 | $\frac{1}{1-x^2}$          |
| $\operatorname{sech}^{-1} x$ | $\frac{-1}{x\sqrt{1-x^2}}$ | $\operatorname{cosech}^{-1} x$ | $\frac{-1}{x\sqrt{1+x^2}}$ |

Ex.  $y = \sinh^{-1}(e^x)$

$$y' = \frac{1}{\sqrt{1+e^{2x}}} \cdot e^x$$

$y = \tanh^{-1}(\ln x^2)$

$$y' = \frac{1}{1-(\ln x^2)^2} \cdot \frac{1}{x^2} \cdot 2x = \frac{2}{x[1-(\ln x^2)^2]}$$

$y = (x^3 + \sqrt{x}) \cdot \cosh^{-1}(e^{x^2})$

$$y' = (x^3 + \sqrt{x}) \cdot \frac{1}{\sqrt{e^{2x^2}-1}} \cdot e^x \cdot 2x + \cosh^{-1}(e^{x^2}) \cdot \left(3x^2 + \frac{1}{2\sqrt{x}}\right)$$

الدوال الغير معروفة: عند التعويض المباشر في الدالة  $\left(\frac{0}{0}, \frac{\infty}{\infty}\right)$  تساوي كمية غير معروفة  
(يجب ان تكون بسط ومقام لا يستخدم قاعدة لوبيتال)

①  $\lim_{x \rightarrow 0} \frac{\sin x}{x} = 1$

②  $\lim_{x \rightarrow 0} \frac{\sin 2x}{3x} = \frac{2}{3} \Rightarrow \lim_{x \rightarrow 0} \frac{\sin \alpha x}{\beta x} = \frac{\alpha}{\beta}$

③  $\lim_{x \rightarrow 0} \frac{\tan x}{x} = 1 \quad \lim_{x \rightarrow 0} \frac{\tan \alpha x}{\beta x} = \frac{\alpha}{\beta}$

④  $\lim_{x \rightarrow 1} \frac{x^2-1}{x-1}$

$\lim_{x \rightarrow 1} \frac{2x}{1} = 2$  or  $\lim_{x \rightarrow 1} \frac{(x-1)(x+1)}{(x-1)} = x+1 = 1+1 = 2$

$$\lim_{x \rightarrow 0} \frac{e^x - 1}{x^3} = \frac{1}{0}$$

$$\frac{e^x}{6x} = \frac{1}{0}$$

$$\lim_{x \rightarrow 0} \frac{e^x}{6} = \frac{e^0}{6} = \frac{1}{6}$$

$$\lim_{x \rightarrow \frac{\pi}{2}} \frac{1 - \sin x}{\cos x} = \frac{1 - \sin x}{\cos x} = \cot x = \cot 90^\circ = 0$$

$$\lim_{x \rightarrow \infty} \frac{\ln x}{x} = \frac{\infty}{\infty}$$

$$\lim_{x \rightarrow \infty} \frac{\frac{1}{x}}{1} = \lim_{x \rightarrow \infty} \frac{1}{x} = \frac{1}{\infty} = 0$$

$$\lim_{x \rightarrow 0} x^{\sin x}$$

$$y = x^{\sin x}$$

$$\ln y = \ln x^{\sin x}$$

$$\ln y = \sin x \ln x$$

$$\frac{y'}{y} = \frac{\sin x}{x} + \ln x \cdot \cos x$$

$$y' = y \left[ \frac{\sin x}{x} + \ln x \cdot \cos x \right]$$

$$e^{\ln y} = e^{\sin x \ln x}$$

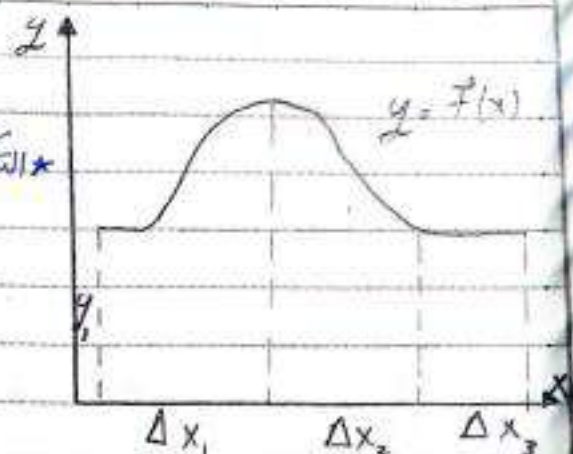
$$y = e^{\sin x \ln x}$$

$$\lim_{x \rightarrow 0} e^{\sin x \ln x}$$

$$\therefore y = 1$$

$$\text{Area} = \int y \cdot dx$$

\* التكامل يعبر عن مساحة الشكل أسفل المنحنى



$$① \int K \cdot dx = K \int dx = Kx + C$$

$$② \int x^n dx = \frac{x^{n+1}}{n+1} + C$$

$$\text{Ex } ① \int \frac{1}{x^5} dx = \int x^{-5} dx = \frac{x^{-4}}{-4} + C$$

$$② \int (2x^4 + 3x^3 - x + 1) dx$$

$$= \int 2x^4 dx + \int 3x^3 dx - \int x dx + \int 1 dx$$

$$= \frac{2x^5}{5} + \frac{3x^4}{4} - \frac{x^2}{2} + x + C$$

$$③ \int \sqrt{x} dx = \int x^{\frac{1}{2}} dx = \frac{2}{3} x^{\frac{3}{2}} + C$$

$$\text{Ex. } \int \frac{T^2 - 2t^4}{t^4} dt = \int \frac{T^2}{t^4} - \frac{2t^4}{t^4} dt$$

$$= \int T^{-2} - 2 dt = -T^{-1} - 2t + C$$

$$\boxed{4} \int (f(x))^n dx \quad f'(x) = \frac{f(x)^{n+1}}{n+1} + C$$

$$\text{Ex. } \int \sqrt{2x-1} dx = \int (2x-1)^{\frac{1}{2}} dx$$

$$= \frac{1}{2} \int (2x-1)^{\frac{1}{2}} \cdot 2 dx = \frac{1}{2} \frac{(2x-1)^{\frac{3}{2}}}{\frac{3}{2}} + C$$

$$\boxed{5} \int (2x+3)^{10} dx = \frac{(2x+3)^{11}}{2} + C$$

"لو"

مشتقة القوس برا

$$\text{Ex. } \int (2x^2+3)^9 \cdot x dx$$

ينزود الأس واحد وتقسيم  
على الأس الجديد

$$= \frac{1}{4} \int (2x^2+3)^9 4x dx$$

$$= \frac{(2x^2+3)^{10}}{10} + C$$

$$\boxed{1} \int \cos x \cdot dx = \sin x + C$$

تكامل الدوال المثلثية

$$\boxed{2} \int \sin x \cdot dx = -\cos x + C$$

$$\boxed{3} \int \sec^2 x \cdot dx = \tan x + C$$

$$\boxed{4} \int \csc^2 x \cdot dx = -\cot x + C$$

$$\boxed{5} \int \sec x \cdot \tan x \cdot dx = \sec x + C$$

$$\boxed{6} \int \csc x \cdot \cot x \cdot dx = -\csc x + C$$

$$\boxed{7} \int \tan x \cdot dx = \int \frac{-\sin x}{\cos x} dx = \ln(\cos x)^{-1} + C$$

or

$$= \ln \sec x + C$$

الأس

$$[8] \int \cot x \, dx = \int \frac{\cos x}{\sin x} \, dx = \ln \sin x + C$$

$$\begin{aligned}
 [9] \int \sec x \, dx &= \int \sec x \cdot \frac{\sec x + \tan x}{\sec x + \tan x} \cdot dx \\
 &= \int \frac{\sec^2 x + \sec x \cdot \tan x}{\sec x + \tan x} \cdot dx \\
 &= \ln(\sec x + \tan x) + C
 \end{aligned}$$

$$\begin{aligned}
 [10] \int \operatorname{cosec} x \, dx &= \int \operatorname{cosec} x \cdot \frac{\operatorname{cosec} x + \cot x}{\operatorname{cosec} x + \cot x} \cdot dx \\
 &= - \int \frac{\operatorname{cosec}^2 x + \operatorname{cosec} x \cot x}{\operatorname{cosec} x + \cot x} \cdot dx \\
 &= - \ln(\operatorname{cosec} x + \cot x) + C
 \end{aligned}$$

$$\begin{aligned}
 [11] \int \frac{\cos x}{\sin^2 x} \cdot dx &= \int \frac{\cos x}{\sin x} \cdot \frac{1}{\sin x} \cdot dx = \int \cot x \cdot \operatorname{cosec} x \cdot dx \\
 &= - \operatorname{cosec} x + C
 \end{aligned}$$

$$[2] \int (x^2 + \sec^2 x) \, dx = \frac{1}{3} x^3 + \tan x + C$$

$$[3] \int \frac{\sin x}{\cos x + 3} \, dx = - \int \frac{\sin x}{\cos x + 3} = - \ln(\cos x + 3) + C$$

$$[11] \int \frac{f'(x)}{f(x)} \cdot dx = \ln f(x) + C$$

البطقة القياسية  
 $\hookrightarrow \ln + \text{البطقة}$

تكمال الدوال الأسية

نفسها إذا كانت متطابقة في مشتقة الأساس

$$\boxed{1} \int e^x dx = e^x + C$$

$$\int e^{f(x)} \cdot f'(x) dx = e^{f(x)} + C$$

$$\boxed{Ex} \int e^{x^2+3} x dx = \frac{1}{2} \int e^{x^2+3} 2x \cdot dx$$
$$= \frac{1}{2} e^{x^2+3} + C$$

$$\boxed{2} \int a^{f(x)} f'(x) \cdot dx = \frac{a^{f(x)}}{\ln a} + C$$

$$\int a^x dx = \frac{a^x}{\ln a} + C$$

$$\boxed{Ex} \int \frac{\ln x}{x} dx = \int \ln x \cdot \frac{1}{x} \cdot dx = \frac{(\ln x)^2}{2} + C$$

$$\boxed{3} \int \frac{dx}{e^{-x}+1} = \int \frac{dx}{e^{-x}+1} \cdot \frac{e^x}{e^x} = \int \frac{e^x}{1+e^x} \cdot dx = \ln(1+e^x) + C$$

$$\boxed{4} \int 2^x \ln 2 dx = \ln 2 \cdot \frac{2^x}{\ln 2} + C = 2^x + C$$

$$\boxed{5} \int 3^{\sin x} \cdot \cos x dx = \frac{3^{\sin x}}{\ln 3} + C$$

تكامل الدوال العكسية

$$\boxed{1} \sin^{-1} x = \int \frac{1}{\sqrt{1-x^2}} \cdot dx$$

$$\boxed{4} \cos^{-1} x = \int \frac{-1}{\sqrt{1-x^2}} \cdot dx$$

$$\boxed{2} \tan^{-1} x = \int \frac{1}{1+x^2} \cdot dx$$

$$\boxed{5} \cot^{-1} x = \int \frac{-1}{1+x^2} \cdot dx$$

$$\boxed{3} \sec^{-1} x = \int \frac{1}{x\sqrt{x^2-1}} \cdot dx$$

$$\boxed{6} \operatorname{cosec}^{-1} x = \int \frac{-1}{x\sqrt{x^2-1}} \cdot dx$$

$$\boxed{\text{Ex}} \quad \textcircled{1} \int \frac{-1}{1+x^2} \cdot dx = -\tan^{-1} x + C$$

$$\textcircled{2} \int \frac{dx}{4x^2+9} = \frac{1}{4} \int \frac{dx}{\frac{9}{4}+x^2} \therefore = \frac{1}{4} \tan^{-1} \frac{x}{\frac{3}{2}} + C$$

$$= \frac{1}{4} \tan^{-1} \frac{2x}{3} + C$$

## lecture seven

$$\boxed{1} \int f(g(x))^n g'(x) dx = \frac{(g(x))^{n+1}}{n+1}$$

(= طريقة استقويض جبري (= ارجع للدالة، لا أصلية)

$$\int (x)^n dx = \frac{(x)^{n+1}}{n+1}$$

Ex:- ①  $\int (x^2+1)^{10} 2x dx$

$$\begin{aligned} u &= x^2 + 1 \\ du &= 2x dx \end{aligned} \Rightarrow \int \frac{u^{11}}{11} + C$$

يعرف

بالستقويض عند  $|u|$  (ارجع للدالة لأصلها).

$$\therefore \frac{(x^2+1)^{11}}{11} + C$$

$$\textcircled{2} \int \sin(x+9) dx$$

$$u = x+9$$

بفرض

$$du = dx$$

$$\int \sin(u) du = -\cos(u) + C$$

$$\hookrightarrow = -\cos(x+9) + C \quad \#$$

[2]

$$\int \sin^2 x \cos x dx = \frac{\sin^3 x}{3} + C$$

دالة في مشتقتها

٩)

$$u = \sin x$$

$$du = \cos x dx$$

$$\Rightarrow \int u^2 \cdot du = \frac{u^3}{3} + C$$

لضع، لوالته لأصلها.

$$= \frac{\sin^3 x}{3} + C \quad \#$$

[3]

$$\int \frac{e^{\sqrt{x}}}{\sqrt{x}} dx = \int e^{\sqrt{x}} \cdot \frac{1}{\sqrt{x}} dx = 2 \int e^{\sqrt{x}} \cdot \frac{1}{2\sqrt{x}} dx$$

$$= 2 e^{\sqrt{x}} + C$$

$$\boxed{4} \int \cos(ax) dx \Rightarrow \frac{\sin ax}{a} + C$$

$$\boxed{5} \int \frac{dx}{a^2 + x^2} = \frac{1}{a} \tan^{-1} \left( \frac{x}{a} \right) + C$$

$$\boxed{6} \int \frac{dx}{\sqrt{a^2 - x^2}} = \sin^{-1} \left( \frac{x}{a} \right) + C$$

$$\boxed{7} \int \frac{-dx}{\sqrt{a^2 - x^2}} = \cos^{-1} \left( \frac{x}{a} \right) + C$$

$$\boxed{8} \int \frac{1}{x\sqrt{x^2 - a^2}} dx = \frac{1}{a} \sec^{-1} \left( \frac{x}{a} \right) + C$$

$$\boxed{9} \int \frac{-dx}{a^2 + x^2} = \frac{1}{a} \cot^{-1} \left( \frac{x}{a} \right) + C$$

$$\boxed{10} \int \frac{-1}{x\sqrt{x^2 - a^2}} dx = \frac{1}{a} \operatorname{Cosec}^{-1} \left( \frac{x}{a} \right) + C$$

$$\underline{\underline{\text{Ex}}} \quad \textcircled{1} \int \frac{dx}{x\sqrt{x^2 - 16}} = \frac{1}{4} \sec^{-1} \left( \frac{x}{4} \right) + C \quad \#$$

لأنهم معامل  $(x^2)$  يساوي الواحد الصحيح  $\Rightarrow$

$$\textcircled{2} \int \frac{dx}{9+4x^2}$$

$$= \frac{1}{4} \int \frac{dx}{\frac{9}{4} + x^2} = \frac{1}{4} \cdot \frac{1}{3/2} \tan^{-1} \frac{x}{3/2} + C$$

$$= \frac{1}{4} \cdot \frac{2}{3} \tan^{-1} \left( \frac{2x}{3} \right) + C$$

$$= \frac{1}{6} \tan^{-1} \left( \frac{2x}{3} \right) + C \quad \#$$

$\boxed{2}$   $\int_1^2 \frac{dx}{9+4x^2} = \frac{1}{6} \tan^{-1} \left( \frac{2x}{3} \right) \Big|_1^2$  التكامل بالحدود

$$= \frac{1}{6} \left[ \left( \tan^{-1} \left( \frac{4}{3} \right) \right) - \left( \tan^{-1} \left( \frac{2}{3} \right) \right) \right] = \dots$$

أوجد المساحة تحت المنحنى (التكامل)

$$\boxed{3} \int_0^{\frac{1}{2}} \frac{1}{\sqrt{1-x^2}} dx = \sin^{-1} x \Big|_0^{\frac{1}{2}} = \frac{\pi}{6} - 0$$

$$= \boxed{\frac{\pi}{6}} \quad \#$$

==

$$\text{III} \int_a^a f(x) dx = 0$$

تلاطم نقطه على المنحنى  
(مقدار ثابت)

ملاحظات

$$\int_0^3 f(x) dx \Rightarrow f(x) \begin{cases} x^2 & x < 2 \\ 3x-2 & x \geq 2 \end{cases}$$

مثال مهم

$$\begin{aligned} &= \int_0^3 f(x) dx = \int_0^2 x^2 dx + \int_2^3 (3x-2) dx \\ &= \left. \frac{1}{3} x^3 \right|_0^2 + \left. \left( \frac{3}{2} x^2 - 2x \right) \right|_2^3 = \dots \end{aligned}$$

$$\textcircled{2} \int \frac{(3x^3 - 4x^2 + 3x)}{x^2 + 1} dx = \int \frac{3x(x^2 + 1) - 4x^2}{(x^2 + 1)} dx$$

$$= \int 3x - \frac{4x^2}{(x^2 + 1)} dx$$

$$= \int 3x dx - \int \frac{4x^2}{x^2 + 1} dx$$

$$= \frac{3}{2} x^2 - \int \frac{4x^2 + 1 - 1}{x^2 + 1} dx$$

$$= \frac{3}{2} x^2 - 4 \left[ \int \frac{x^2 + 1}{x^2 + 1} dx - \int \frac{1}{x^2 + 1} dx \right]$$

$$= \frac{3}{2} x^2 - 4x + 4 \tan^{-1}(x) + C \quad \#$$

$$\begin{aligned}
 \textcircled{3} \quad \int \sin^4 x \cos^3 x \cdot dx &= \int \sin^4 x \cos^2 x \cos x \, dx \\
 &= \left[ \int \sin^4 x \cdot \cos x \cdot (1 - \sin^2 x) \, dx \right] \\
 &= \left[ \int \sin^4 x \cdot \cos x - \sin^6 x \cdot \cos x \right] dx \\
 &= \left[ \int \sin^4 x \cdot \cos x \, dx - \int (\sin x)^6 \cos x \, dx \right] \\
 &= \frac{\sin^5 x}{5} - \frac{\sin^7 x}{7} + C \quad \#
 \end{aligned}$$

اثبات قانون التفاضل لـ  $(\tan^{-1})$

$$\frac{d}{dx} \tan^{-1} \left( \frac{x}{a} \right) = \frac{1}{1 + \frac{x^2}{a^2}} \cdot \frac{1}{a} = \frac{1}{a^2 + \frac{x^2}{a^2}} \cdot \frac{1}{a}$$

$$= \frac{a^2}{a^2 + x^2} \cdot \frac{1}{a} = \frac{a}{a^2 + x^2}$$

$$\therefore \frac{d}{dx} \tan^{-1} \left( \frac{x}{a} \right) = \frac{1}{a^2 + x^2} \Rightarrow \frac{1}{a} \tan^{-1} \left( \frac{x}{a} \right)$$

$$\frac{1}{a} \tan^{-1} \left( \frac{x}{a} \right) = \int \frac{dx}{a^2 + x^2} \quad \#$$

==

## التكامل بالتجزئ

$$\int f(x) \cdot g(x) dx$$

$$u = f(x)$$

$$dv = g(x) dx$$

$$du = f'(x) dx$$

$$v = \int g(x) dx$$

$$I = f(x) \cdot \int g(x) dx - \int v du$$

$$\hookrightarrow I = f(x) \cdot \int g(x) dx - \int g(x) f'(x) dx$$

$$I = uv - \int v du$$

Ex:  $\int x e^x dx$

$$u = x$$

$$dv = e^x dx$$

$$du = dx$$

$$v = e^x$$

$$I = uv - \int v du = x e^x - \int e^x dx$$

$$= \boxed{x e^x - e^x + C} \quad \#$$

ex:

$$\int e^x \cdot x^2 dx$$

$$u = x^2$$

$$dv = e^x dx$$

$$du = 2x dx$$

$$v = e^x$$

$$I = x^2 e^x - 2 \int x e^x dx$$

$$\therefore I = x^2 e^x - 2 II$$

$$\hookrightarrow II = \int x e^x dx = x e^x - e^x + C$$

$$\therefore I = x^2 e^x - 2 [x e^x - e^x] + C \quad \#$$

[2]  $\int x^4 e^x dx$

( $x^4$ ) ٤ مرّة  
تفاضل و تكامل

$$\begin{array}{rcl}
 x^4 & \xrightarrow{+} & e^x \\
 4x^3 & \xrightarrow{-} & e^x \\
 12x^2 & \xrightarrow{+} & e^x \\
 24x & \xrightarrow{-} & e^x \\
 24 & \xrightarrow{+} & e^x \\
 0 & \xrightarrow{-} & e^x
 \end{array}$$

$$I = x^4 e^x - 4x^3 e^x + 12x^2 e^x - 24x e^x + 24 e^x + C$$

[3]  $\int x^4 e^{-x} dx$

$$\begin{array}{rcl}
 x^4 & \xrightarrow{+} & e^{-x} \\
 4x^3 & \xrightarrow{-} & -e^{-x} \\
 12x^2 & \xrightarrow{+} & e^{-x} \\
 24x & \xrightarrow{-} & -e^{-x} \\
 24 & \xrightarrow{+} & e^{-x} \\
 0 & \xrightarrow{-} & -e^{-x}
 \end{array}$$

$$I = -x^4 e^{-x} - 4x^3 e^{-x} - 12x^2 e^{-x} - 24x e^{-x} - 24 e^{-x} + C$$

(4)

$$\int x \cos x \, dx$$

$$u = x$$

$$du = dx$$

$$dv = \cos x \, dx$$

$$v = \sin x$$

⊖

$$I = x \sin x - \int \sin x \, dx$$

$$= \underline{x \sin x + \cos x + C} \quad \#$$

(5)

$$\int x^2 \cos x \, dx$$

$$u = x^2$$

$$du = 2x \, dx$$

$$dv = \cos x \, dx$$

$$v = \sin x$$

⊖

$$I = x^2 \sin x - 2 \int x \sin x \, dx$$

$$= x^2 \sin x - 2 II$$

↙

$$II = \int x \sin x \, dx$$

⇒

$$u = x$$

$$du = dx$$

$$dv = \sin x \, dx$$

$$v = -\cos x$$

$$II = -x \cos x + \int \cos x \, dx$$

$$II = -x \cos x + \sin x + C$$

$$\therefore I = x^2 \sin x - 2 \left[ -x \cos x + \sin x \right] + C$$

$$I = x^2 \sin x + 2 \left[ x \cos x + \sin x \right] + C \quad \#$$

$$\boxed{1} \int x^4 \cos x \, dx$$

|         |           |           |
|---------|-----------|-----------|
| $x^4$   | $\oplus$  | $\cos x$  |
| $4x^3$  | $\ominus$ | $\sin x$  |
| $12x^2$ | $\oplus$  | $-\cos x$ |
| $24x$   | $\ominus$ | $-\sin x$ |
| $24$    | $\oplus$  | $\cos x$  |
| $0$     | $\ominus$ | $\sin x$  |

$$I = x^4 \sin x + 4x^3 \cos x - 12x^2 \sin x - 24x \cos x + 24 \sin x + C \quad \#$$

$$\boxed{2} \int x \ln x \, dx$$

$$u = \ln x \quad dv = x \, dx$$

$$du = \frac{1}{x} dx \quad v = \frac{x^2}{2}$$

$$\therefore I = \frac{x^2}{2} \ln x - \frac{1}{2} \int x \, dx$$

$$= \frac{x^2}{2} \ln x - \frac{1}{2} \left[ \frac{x^2}{2} \right]$$

$$I = \frac{x^2}{2} \ln x - \frac{x^2}{4} + C \quad \#$$

$$\boxed{3} \int \ln x \, dx$$

$$u = \ln x \quad dv = dx$$

$$du = \frac{1}{x} dx \quad v = x$$

$$I = x \ln x - \int dx = x \ln x - x + C \quad \#$$

$$\boxed{4} \int \ln x \, dx$$

$$u = \ln x$$

$$dv = dx$$

$$du = \frac{1}{x} dx$$

$$v = x$$

$$I = x \ln x - \int dx = x \ln x - x + C \quad \#$$

$$\int x^n \ln x \, dx$$

$$u = \ln x$$

$$dv = x^n dx$$

$$du = \frac{1}{x} dx$$

$$v = \frac{x^{n+1}}{n+1}$$

$$I = uv - \int v du = \frac{x^{n+1}}{n+1} \ln x - \frac{1}{n+1} \int \frac{1}{x} \cdot x^{n+1} dx$$

$$= \frac{x^{n+1}}{n+1} \ln x - \frac{1}{n+1} \int x^n dx$$

$$= \left[ \frac{x^{n+1}}{n+1} \ln x - \frac{1}{(n+1)^2} x^{n+1} + C \right] \quad \#$$

$$\boxed{5} \int \tan^{-1} x \, dx$$

$$u = \tan^{-1} x$$

$$dv = dx$$

$$du = \frac{1}{1+x^2} dx$$

$$v = x$$

$$I = x \tan^{-1} x - \int \frac{x}{1+x^2} dx$$

$$= x \tan^{-1} x - \frac{1}{2} \ln(1+x^2) + C \quad \#$$

$$\textcircled{1} \int x \tan^{-1} x \, dx$$

$$u = \tan^{-1} x$$

$$du = \frac{1}{1+x^2}$$

$$dv = x \, dx$$

$$v = \frac{x^2}{2}$$

$$I = \frac{x^2}{2} \tan^{-1} x - \frac{1}{2} \int \frac{x^2 - 1 + 1}{1+x^2} \, dx$$

$$I = \frac{x^2}{2} \tan^{-1} x - \frac{1}{2} \left[ \int dx - \int \frac{1}{1+x^2} \, dx \right]$$

$$= \frac{x^2}{2} \tan^{-1} x - \frac{1}{2} [x - \tan^{-1} x] + C$$

$$= \frac{\tan^{-1} x}{2} (x^2 + 1) - \frac{1}{2} x + C \quad \#$$

الأسور الجزئية

$$\int \frac{A}{ax+b} \, dx = A \int \frac{dx}{ax+b} = \frac{A}{a} \int \frac{a \, dx}{ax+b}$$

$$= \frac{A}{a} \ln(ax+b) + C \quad \#$$

Ex.  $\int \frac{(x+1)}{2x^2+4x+5} \, dx$

$$= \frac{1}{4} \int \frac{4(x+1)}{2x^2+4x+5} \, dx = \frac{1}{4} \ln(2x^2+4x+5) + C$$

# ① طريقة التفاضل الجزئية

١- اعلان احدى البسط واعلان احدى المقام

٢- احدى البسط اقل من احدى المقام (تلاعل فعلت).

$$\int \frac{x-2}{x^2-2x-3} dx = \frac{(x-2)}{(x+1)(x-3)}$$

$$= \frac{A}{(x+1)} + \frac{B}{(x-3)} \quad \text{ببوصلة المتفاوتات}$$

$$= \frac{A(x-3) + B(x+1)}{(x+1)(x-3)}$$

بالاستعانة على المقام

$$x-2 = A(x-3) + B(x+1)$$

بأخذ الصفر المقام

$$x=3 \quad 1 = 0 + 4B \quad \therefore B = \frac{1}{4}$$

$$x=-1 \quad -3 = -4A + 0 \quad A = \frac{3}{4}$$

$$I = \int \frac{3/4}{(x+1)} + \int \frac{1/4}{x-3}$$

$$= \frac{3}{4} \int \frac{1}{x+1} dx + \frac{1}{4} \int \frac{1}{x-3} dx$$

$$= \frac{3}{4} \ln(x+1) + \frac{1}{4} \ln(x-3) + C$$

\* طريقة الكسر الجزئي

$$\int \frac{4x}{(x+1)(x-1)^2} dx$$

درجة البسط < درجة المقام (غير فطر)  
درجة البسط > المقام (فطر)

$$= \frac{A}{(x+1)} + \frac{B}{(x-1)^2} + \frac{C}{(x-1)}$$

تبويب المقامات

$$= \frac{A(x-1)^2 + B(x+1) + C(x-1)(x+1)}{(x+1)(x-1)^2}$$

مقام البسط

$$\text{بالضرب في المقام} \quad \frac{A(x-1)^2 + B(x+1) + C(x-1)(x+1)}{(x+1)(x-1)^2} \cdot (x+1)(x-1)^2$$

$$= A(x-1)^2 + B(x+1) + C(x-1)(x+1)$$

لضمان المقام لقيمة (x) لإيجاد A, B, C

$$x = 1, \quad x = -1$$

$$x = 1 \Rightarrow B = 2$$

$$x = -1 \Rightarrow A = -1$$

لجمع المعادلات لإيجاد قيمة (C)

$$4x = A(x^2 - 2x + 1) + B(x+1) + C(x^2 - 1)$$

$$0 = A + B - C$$

$$0 = -1 + 2 - C$$

$$\therefore \underline{C = 1}$$

Subject: \_\_\_\_\_

Date: / /

$$A = -1, \quad B = 2, \quad C = 1$$

$$= \frac{A}{x+1} + \frac{B}{(x-1)^2} + \frac{C}{(x-1)}$$

$$= \frac{-1}{x+1} + \frac{2}{(x-1)^2} + \frac{1}{(x-1)}$$

$$= - \int \frac{1}{x+1} dx + 2 \int \frac{1}{(x-1)^2} dx + \int \frac{1}{(x-1)} dx \quad \text{. دالة}$$

$$= -\ln(x+1) - 2(x-1)^{-1} + \ln(x-1) \quad \#$$