

(1)

الكسور الجزئية

نستخدم الكسور الجزئية لتكامل إبدال النسبية (الكسرية) ولتربيع صورة $\frac{P(x)}{q(x)}$ حيث
 لك $P(x)$ كثيران الحدود من المتغير (x) وتسمى الكسور الجزئية بالكسور البغلي (الحقيق)
 إذا كانت درجة البسط أقل من درجة المقام. وتسمى بالكسور غير البغلي إذا كانت درجة
 البسط أكبر من درجة المقام أو تساوي درجة المقام وطريقة الكسور الجزئية تتعامل مع
 تحليل الكسر البغلي مباشرة. في حالة غير البغلي يتم استخدام الصيغة المأولة

(مثال) أوجد لتكامل $I = \int \frac{5x+2}{3x^2+4x-4} dx$

(الحل)

$$\frac{5x+2}{3x^2+4x-4} = \frac{5x+2}{(x+2)(3x-2)} = \frac{A}{x+2} + \frac{B}{3x-2} \quad \text{اطلع الجال} \quad x = -2 \quad x = \frac{2}{3}$$

ضرب بالمأولة $(x+2)(3x-2)$

$$5x+2 = (3x-2)A + (x+2)B$$

$$x = \frac{2}{3}$$

$$5 \times \frac{2}{3} + 2 = (3 \times \frac{2}{3} - 2)A + (\frac{2}{3} + 2)B$$

$$\frac{10}{3} + 2 = 0 +$$

$$\frac{16}{3} = \frac{8}{3} B \rightarrow B = \frac{16}{3} \times \frac{3}{8} = 2$$

$$x = -2$$

$$-10+2 = (-6-2)A + (-2+2)B$$

$$-8 = -8A + 0$$

$$A = 1$$

$$\int \frac{5x+2}{3x^2+4x-4} dx = \int \frac{1}{x+2} dx + \left(\frac{2}{3}\right) \int \frac{\frac{3}{2}x-2}{3x-2} dx$$

$$\frac{d}{dx}(3x-2) = 3$$

$$= \left[\ln(x+2) + \frac{2}{3} \ln(3x-2) + C \right]$$

Note

لو البسط أقل من المقام

$\tan^{-1}()$

المقام يتحلل ويتم حلة بالكسور الجزئية

(2)

$$\int \frac{1}{x \sqrt{1+x^3}} dx$$

$$\text{let } y = 1+x^3 \rightarrow 2y dy = 3x^2 dx \rightarrow dx = \frac{2y}{3x^2} dy$$

$$\int \frac{1}{x \sqrt{y^3}} \cdot \frac{2y}{3x^2} dy \rightarrow \int \frac{2y}{3x^3 y} dy \quad x^3 = y-1$$

$$\frac{2}{3} \int \frac{1}{y^2-1} dy \xrightarrow[\text{الجزئية}]{\text{مستعمل باكور}} \frac{2}{3} \int \frac{1}{(y+1)(y-1)} dy$$

$$\frac{\frac{2}{3}}{(y+1)(y-1)} = \frac{A}{y+1} + \frac{B}{y-1}$$

$$\boxed{\text{if } y=-1} \quad \boxed{y=1}$$

لنجد المعادلة $(y+1)(y-1)$

$$\frac{2}{3} = (y-1)A + (y+1)B$$

$$\text{if } y=-1 \quad \frac{2}{3} = (-1-1)A \rightarrow \frac{2}{3} = -2A \rightarrow A = \frac{2}{3} \times \frac{1}{-2} = -\frac{1}{3}$$

$$\text{if } y=1 \quad \frac{2}{3} = (1+1)B \rightarrow \frac{2}{3} = 2B \rightarrow B = \frac{1}{3}$$

$$\int \frac{\frac{2}{3}}{y^2-1} dy = -\frac{1}{3} \int \frac{1}{y+1} + \frac{1}{3} \int \frac{1}{y-1}$$

$$= -\frac{1}{3} \ln(y+1) + \frac{1}{3} \ln(y-1)$$

$$y = \sqrt{1+x^3}$$

$$= -\frac{1}{3} \ln(\sqrt{1+x^3}+1) + \frac{1}{3} \ln(\sqrt{1+x^3}-1)$$

فرحالة انكسر إضير الفعلى (البسيط أكبر من أوساوس المقام)

(3)

لتم استخدام إقسمة المطولة انكسر الفير فعلى

$$\int \dots = \text{الباقي} + \frac{\text{الناتج}}{\text{المقسوم عليه}}$$

مثال أوجد $I = \int \frac{2x^3 + 2x + 1}{x^2 + 1} dx$ (الحل)

عندى هنا البسيط أكبر من المقام فبالفقال عندى انكسر غير فعلى فاستخدم القسمة المطولة

$$\begin{array}{r} 2x \\ x^2 + 1 \overline{) 2x^3 + \dots + 2x + 1} \\ \underline{2x^3 + \dots + 2x} \\ 1 \end{array}$$

الناتج أقل من المقسوم عليه
القسمة انتهت

$$\int \frac{2x^3 + 2x + 1}{x^2 + 1} dx = \int 2x dx + \int \frac{1}{x^2 + 1} dx$$

$$= x^2 + \tan^{-1} x + C$$

مثال أوجد $I = \int \frac{3x^4 - 3x^2 - x}{x^2 - 1} dx$

$$\begin{array}{r} 3x^2 \\ x^2 - 1 \overline{) 3x^4 - \dots - 3x^2 - x} \\ \underline{3x^4 - \dots - 3x^2} \\ -x \end{array}$$

أقل من المقسوم عليه
القسمة انتهت

$$\int \frac{3x^4 - 3x^2 - x}{x^2 - 1} dx = \int 3x^2 dx - \frac{1}{2} \int \frac{2x}{x^2 - 1} dx$$

$$= x^3 - \frac{1}{2} \ln(x^2 - 1) + C$$

(٧) الحالة الثانية إذا كانت عوامل المقام من الدرجة الأولى ويكون فيها أكثر من مرة

$$g(x) = (ax+b)^n = \frac{A}{(ax+b)} + \frac{B}{(ax+b)^2} + \dots + \frac{D}{(ax+b)^n}$$

درجة أولى
∴ نضع أبسط
أقل من المقام
وهو ثابت

$$I = \int \frac{x^5 + 2x^3 - 1}{x(x-1)^3} dx$$

مثال

$$\int \frac{x^5 + 2x^3 - 1}{x(x-1)^3} dx = \frac{A}{x} + \frac{B}{(x-1)} + \frac{C}{(x-1)^2} + \frac{D}{(x-1)^3}$$

$$\begin{array}{l|l} x=0 & x=-1 \\ x=1 & x=2 \end{array}$$

بضرب الجاهلة $x(x-1)^3$

$$x^5 + 2x^3 - 1 = (x-1)^3 A + x(x-1)^2 B + x(x-1)C + xD$$

$$\text{if } x=0 \rightarrow -1 = (x-1)^3 A \rightarrow -1 = -A \rightarrow A=1$$

$$x=1 \rightarrow 1+2-1 = D \rightarrow D=2$$

$$x=-1 \rightarrow -1-2-1 = (-1-1)^3 A - (-1-1)^2 B - (-1-1)C - D$$

$$-4 = -8(1) - 4B + C - 2$$

$$\boxed{-4B + C = 6} \rightarrow (1)$$

$$x=2 \rightarrow 32+16-1 = (2-1)^3 A + 2(2-1)^2 B + 2(2-1)C + 2D$$

$$47 = A + 2B + 2C + 2D = 1 + 2B + 2C + 4$$

$$\boxed{2B + 2C = 42} \rightarrow (2)$$

$$8B - 2C = -12$$

$$10B = 30 \rightarrow B=3$$

بضرب (1) بـ (-2)

الحالة الثانية: عوامل وفقاً لمبرهنه الأول مكررة

$$\frac{x^2+1}{(x-1)^2(x+1)^2} = \frac{A}{(x-1)} + \frac{B}{(x-1)^2} + \frac{C}{(x+1)} + \frac{D}{(x+1)^2}$$

نص: إذا $(x-1)$ $(x+1)$ $(x-1)^2$ $(x+1)^2$

$$x^2+1 = (x-1)(x+1)^2A + (x+1)^2B + (x-1)^2(x+1)C + (x-1)^2D$$

إذا $x=-1$ $2 = (-1-1)^2D \rightarrow 2 = 4D \rightarrow D = \frac{1}{2}$

إذا $x=1$ $2 = (2)^2B \rightarrow 2 = 4B \rightarrow B = \frac{1}{2}$

لنعين لثوابت

$$x^2+1 = (x-1)(x^2+2x+1)A + (x^2+2x+1)B + (x^2-2x+1)(x+1)C + (x^2-2x+1)D$$

$$= \underline{Ax^3} + \underline{2Ax^2} + \underline{Ax} - \underline{Ax^3} - \underline{2Ax} - \underline{A} + \underline{Bx^2} + \underline{2Bx} + \underline{B} + \underline{Cx^3} - \underline{2Cx^2} + \underline{Cx} + \underline{Cx^2} - \underline{2Cx} + \underline{C} + \underline{Dx^2} - \underline{2Dx} + \underline{D}$$

معامل x^3
= 0 $0 = A + C \rightarrow A = -C$

معامل x^2
= 1 $1 = 2Ax^2 + Ax^2 + Bx^2 - 2Cx^2 + Cx^2 + Dx^2$

$$1 = 3Ax^2 + Bx^2 - Cx^2 + Dx^2$$

$$1 = 3C + \frac{1}{2} - C + \frac{1}{2} \rightarrow -4C = 0 \rightarrow C = 0, A = 0$$

معامل x
 $x=0$ $0 = A - 2A + 2B + C - 2C - 2D$

$$0 = -A + C - 2D \rightarrow A = 0, C = 0$$

$$1 = -A + B + C + D$$

$$1 = -A + \frac{1}{2} + C + \frac{1}{2} \rightarrow A = 0, C = 0$$

$$\int \frac{x^2+1}{(x-1)^2(x+1)^2} dx = \int \frac{0}{x-1} dx + \frac{1}{2} \int \frac{1}{(x-1)^2} dx + \int \frac{0}{x+1} dx + \frac{1}{2} \int \frac{1}{(x+1)^2} dx$$

$$= \frac{1}{2} \cdot \frac{1}{(x-1)} + \frac{1}{2} \cdot \frac{1}{x+1} + C$$

$$= \frac{1}{2} \left(\frac{(x+1) + (x-1)}{(x-1)(x+1)} \right) = \frac{1}{2} \cdot \frac{2x}{(x-1)^2} + C$$

(b)

مثال أوجد التكاملات التالية

1- $\int \frac{(3x+13)}{x^2+4x+3} dx$

السطح اقل من المقام
لذلك كامل فعلى

أضمار المقام

$$\frac{3x+13}{(x+1)(x+3)} = \frac{A}{x+1} + \frac{B}{x+3}$$

if $x = -1$ $x = -3$

نضرب في $(x+1)(x+3)$

$$3x+13 = (x+3)A + (x+1)B$$

if $x = -1$ $-3+13 = (-1+3)A \rightarrow 10 = 2A \rightarrow A = 5$

if $x = -3$ $-9+13 = (-3+1)B \rightarrow 4 = -2B \rightarrow B = -2$

$$\int \frac{3x+13}{x^2+4x+3} = 5 \int \frac{1}{x+1} dx - 2 \int \frac{1}{x+3} dx$$
$$= 5 \ln(x+1) - 2 \ln(x+3) + C$$

2- $\int \frac{4x+1}{x^3-x^2-2x} dx$

السطح اقل من المقام
لذلك كامل فعلى

أضمار المقام

$$\int \frac{4x+1}{x(x^2-x-2)} dx = \int \frac{4x+1}{x(x-2)(x+1)} dx$$

if $x_1 = 0$ $x_2 = 2$ $x_3 = -1$

نضرب في $x(x-2)(x+1)$

$$\frac{4x+1}{x(x-2)(x+1)} = \frac{A}{x} + \frac{B}{x-2} + \frac{C}{x+1}$$

$$4x+1 = (x-2)(x+1)A + x(x+1)B + x(x-2)C$$

if $x = 0$ $(4 \cdot 0) + 1 = (0-2)(0+1)A \rightarrow 1 = -2A \rightarrow A = -\frac{1}{2}$

if $x = 2$ $(2 \cdot 4) + 1 = 2(2+1)B \rightarrow 9 = 6B \rightarrow B = \frac{3}{2}$

if $x = -1$ $(4 \cdot (-1) + 1) = -1(-1-2)C \rightarrow -3 = 3C \rightarrow C = -1$

$$\int \frac{4x+1}{x^3-x^2-2x} dx = -\frac{1}{2} \int \frac{1}{x} dx + \frac{3}{2} \int \frac{1}{x-2} dx - \int \frac{1}{x+1} dx$$
$$= -\frac{1}{2} \ln x + \frac{3}{2} \ln(x-2) - \ln(x+1) + C$$

(7)

$$\int \frac{5x-1}{x^3(2x-1)^2} dx \rightarrow x=0 \quad x=\frac{1}{2}$$

$$\frac{5x-1}{x^3(2x-1)^2} = \frac{Ax+B}{x^2} + \frac{C}{2x-1} + \frac{D}{(2x-1)^2} \rightarrow \text{خبر با مساوی } x^3(2x-1)^2$$

$$5x-1 = (2x-1)^2 Ax + B + x^3(2x-1)C + x^2 D$$

$$\text{If } x=0 \quad -1 = (2 \cdot 0 - 1)^2 A(0) + B \rightarrow -1 = B$$

$$\text{If } x=\frac{1}{2} \quad \frac{5}{2}-1 = \left(\frac{1}{2}\right)^2 D \rightarrow \frac{3}{2} = \frac{1}{4} D \rightarrow D=6$$

$$5x-1 = (4x^2-4x+1)(Ax+B) + 2Cx^3 - Cx^2 + Dx^2$$

$$5x-1 = 4Ax^3 - 4Ax^2 + Ax + 4Bx^2 - 4Bx + B + 2Cx^3 - Cx^2 + Dx^2$$

$$\begin{array}{l} \text{مقابل} \\ x^3 \\ (=0) \end{array} \quad 0 = 4A + 2C$$

$$\begin{array}{l} \text{مقابل} \\ x^2 \\ (=0) \end{array} \quad 0 = -4A + 4B - C + D \rightarrow 0 = -4A - 4 - C + 6 \rightarrow -2 = -4A - C$$

$$\begin{array}{l} \text{مقابل} \\ x \\ (=5) \end{array} \quad 5 = A - 4B \rightarrow 5 = A + 4 \rightarrow A=1 \quad C=-2$$

$$\int \frac{5x-1}{x^3(2x-1)^2} = \frac{1}{2} \int \frac{2x-1}{x^2} dx - \int \frac{2}{2x-1} dx + \int \frac{1}{(2x-1)^2} dx$$

$$= \frac{1}{2} \ln(x^2) - \ln(2x-1) + 6(2x-1)^{-1} + C$$

$$= \frac{1}{2} \ln x^2 - \ln(2x-1) + \frac{6}{2x-1} + C$$

$$\int \frac{5x-1}{x^3(2x-1)^2} dx = \frac{1}{2} \ln \frac{x^2}{2x-1} + \frac{6}{2x-1} + C$$

إكمال المربع

لإيجاد إنتكاملات ax^2+bx+c اوعلى $\sqrt{ax^2+bx+c}$ فنستخدم طريقة إكمال المربع

مثال (1) $I = \int \frac{dx}{\sqrt{2+3x-2x^2}}$

$$\int \frac{dx}{\sqrt{2(1+\frac{3}{2}x-x^2)}} = \frac{1}{\sqrt{2}} \int \frac{1}{\sqrt{1+\frac{3}{2}x-x^2}}$$

$$\frac{1}{\sqrt{2}} \int \frac{1}{\sqrt{1+\frac{3}{2}x-x^2+(\frac{3}{4})^2-(\frac{3}{4})^2}} = \frac{1}{\sqrt{2}} \int \frac{1}{\sqrt{\frac{25}{16}-(x-\frac{3}{4})^2}}$$

$$\begin{aligned} & \frac{3}{2}x - x^2 \\ & (\frac{3}{4} - x)^2 \\ & (\frac{9}{16} - \frac{3}{2}x + x^2) \\ & - (\frac{3}{4} - x)^2 \end{aligned}$$

التعويض $dy = dx$ $y = x - \frac{3}{4}$

$$\frac{1}{\sqrt{2}} \int \frac{1}{\sqrt{\frac{25}{16} - y^2}} = \frac{1}{\sqrt{2}} \sin^{-1} \frac{y}{\frac{5}{4}} = \frac{1}{\sqrt{2}} \sin^{-1} \frac{x-\frac{3}{4}}{\frac{5}{4}} + c$$

$$I = \int \frac{dx}{2x^2 - 5x + 3}$$

$$\frac{1}{2} \int \frac{1}{x^2 - \frac{5}{2}x + \frac{3}{2}} = \frac{1}{2} \int \frac{1}{x^2 - \frac{5}{2}x + \frac{25}{16} + \frac{3}{2} - \frac{25}{16}} = \frac{1}{2} \int \frac{1}{(x - \frac{5}{4})^2 - \frac{1}{16}} dx$$

$$\begin{aligned} & (x - \frac{5}{4})^2 \\ & = x^2 - \frac{5}{2}x + \frac{25}{16} \end{aligned}$$

let $y = x - \frac{5}{4} \rightarrow dy = dx$

$$\frac{1}{2} \int \frac{1}{y^2 - \frac{1}{16}} dy = -\frac{1}{2} \int \frac{1}{\frac{1}{16} - y^2} dy = -\frac{1}{2} \tanh^{-1} \frac{y}{\frac{1}{4}} + c$$

$$a = \frac{1}{4}$$

$$y \cdot -\frac{1}{2} \tanh^{-1} 4y + c$$

$$= -2 \tanh^{-1} 4y + c$$

$$I = \int \frac{3x-5}{\sqrt{x^2-4x+5}} dx = \int \frac{2x-4}{\sqrt{x^2-4x+5}} dx + \int \frac{1}{\sqrt{x^2-4x+5}} dx$$

$$\boxed{2x-4}$$

$$3x-5 = \frac{3}{2} \left(2x - \frac{10}{3} \right)$$

$$= \frac{3}{2} \left(2x - 4 + 4 - \frac{10}{3} \right)$$

$$= \frac{3}{2} (2x-4) + \frac{3}{2} \left(\frac{2}{3} \right)$$

$$= \frac{3}{2} (2x-4) + 1$$

$$I = \frac{3}{2} \int \frac{2x-4}{\sqrt{x^2-4x+5}} dx + \int \frac{1}{\sqrt{x^2-4x+5}} dx$$

$$= 3 \sqrt{x^2-4x+5} + \int \frac{1}{\sqrt{x^2-4x+5}} dx$$

$$I' = \int \frac{1}{\sqrt{(x-2)^2+1}} dx$$

$$= \sinh^{-1}(x+2)$$

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المسألة الأولى (١١) (٢٠)

« (١١) - (١١) »

المسألة

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في حالة التفاضل الجزئي

$$\frac{d}{dx} (\pi) = 0$$

$$\frac{d}{dx} [\sqrt{2}] = 0$$

$$\frac{d}{dx} (x^3) = 3x^2$$

$$\frac{d^2}{dx^2} (x^3) = 6x$$

$$\frac{d}{dx} [(x^2+1)^3] = 3(x^2+1)^2 \cdot 2x = 6x(x^2+1)^2$$

$$\frac{d}{dx} [(x^2+2x)^4] = 4(x^2+2x)^3 (2x+2) = 8(x+1)(x^2+2x)^3$$

$$\frac{d}{dx} [x^3(x+1)^2] = x^3 \cdot 2(x+1) + 3x^2(x+1)^2 = 2x^3(x+1) + 3x^2(x+1)^2$$

$$\frac{d}{dx} \left[\frac{3x^2}{(x+1)} \right] = \frac{6x(x+1) - 3x^2}{(x+1)^2} = \frac{3x^2 + 6x}{(x+1)^2}$$

$$\frac{d}{dx} \left[\frac{3}{(x^2+1)} \right] = \frac{0 - 3 \cdot 2x}{(x^2+1)^2} = \frac{-6x}{(x^2+1)^2}$$

$$\frac{d}{dx} \left[\frac{1}{\sqrt{x}} \right] = \frac{d}{dx} (x^{-1/2}) = -\frac{1}{2} x^{-3/2} = -\frac{1}{2x^{3/2}}$$

$$\frac{d}{dx} [\sqrt{3x^2+2}] = \frac{6x}{2\sqrt{3x^2+2}} = \frac{3x}{\sqrt{3x^2+2}}$$

$$\frac{d}{dx} \left[\frac{1}{\sqrt[3]{x^2}} \right] = \frac{d}{dx} [x^{-2/3}] = -\frac{2}{3} x^{-5/3} = -\frac{2}{3x^{5/3}}$$

$$\frac{d}{dx} [5^{x^2}] = 5^{x^2} \ln(5) \cdot 2x = 2x \ln(5) \cdot 5^{x^2}$$

$$\frac{d}{dx} [3^x] = 3^x \ln(3)$$

$$\frac{d}{dx} [e^{x^2}] = 2x e^{x^2}$$

$$\frac{d}{dx} \ln(x^2+1) = \frac{2x}{x^2+1}$$

$$\frac{d}{dx} [\cos(x^2)] = -\sin(x^2) \cdot 2x = -2x \sin(x^2)$$

$$\frac{d}{dx} [\sin(e^{x^2})] = \cos(e^{x^2}) \cdot 2x e^{x^2} = 2x e^{x^2} \cos(e^{x^2})$$

$$\frac{d}{dx} [\tan(x^3)] = 3x^2 \sec^2(x^3)$$

$$\frac{d}{dx} \sec(\ln x^2) = \sec(\ln x^2) \cdot \tan(\ln x^2) \cdot \frac{2x}{x^2} = \frac{2}{x} \sec(\ln x^2) \cdot \tan(\ln x^2)$$

الآن ننتقل إلى

$$\begin{aligned} \text{[1]} \quad \frac{d}{dx} [\sin^n(u)] &= \frac{d}{dx} [\sin(u)]^n \\ &= n [\sin(u)]^{n-1} \cdot \cos(u) \cdot u' \end{aligned}$$

والآن ننتقل إلى المثال الثاني، وهو إيجاد مشتق الدالة $y = \tan(\sin(x))$

$$\begin{aligned} \text{[2]} \quad y &= \tan(\sin(x)) \\ y' &= \sec^2(\sin(x)) \cdot \cos(x) \\ &= \cos(x) \cdot \sec^2(\sin(x)) \end{aligned}$$

(نلاحظ أن الدالة y هي دالة مركبة، لذلك نستخدم قاعدة السلسلة)

[1] Find $\frac{dy}{dx}$ if $y = -3x^{12}$

$$\frac{dy}{dx} = -36x^{11}$$

[2] Find $\frac{dy}{dx}$ if $y = 3x^8 + 2x + 1$

$$\frac{dy}{dx} = 24x^7 + 2$$

[3] Find $f'(x)$ if $f(x) = \sqrt{x} + \frac{1}{x}$

$$f(x) = \sqrt{x} + x^{-1} \Rightarrow f'(x) = \frac{1}{2\sqrt{x}} - \frac{1}{x^2}$$

[4] Find $f'(x)$ if $f(x) = -3x^{-8} + 2\sqrt{x}$

$$f'(x) = 24x^{-9} + \frac{2}{2\sqrt{x}} = \frac{24}{x^9} + \frac{1}{\sqrt{x}}$$

[5] Find $f'(x)$ if $f(x) = xe^x + \frac{1}{x\sqrt{10}}$

$$f'(x) = e^x e^x - \sqrt{10} x^{-\sqrt{10}-1}$$

[6] $f'(x)$ if find $y'(1)$ if $y = \frac{x^{3/2} + 2}{x}$

$$f(x) = \frac{\frac{3}{2}x^{1/2} \cdot x - (x^{3/2} + 2)}{x^2} = \frac{\frac{3}{2}x^{3/2} - x^{3/2} - 2}{x^2}$$

$$= \frac{\frac{1}{2}x^{3/2} - 2}{x^2}$$

$$\therefore f'(1) = \left[\frac{-3}{2} \right]$$

[7] Find $\frac{d^2y}{dx^2}$ if $y = 7x^3 - 5x^2 + x$

$$\frac{dy}{dx} = 21x^2 - 10x + 1$$

$$\frac{d^2y}{dx^2} = 42x - 10$$

[8] Find $\frac{d^2y}{dx^2}$ if $y = \frac{x+1}{x}$

$$\frac{dy}{dx} = \frac{x - x - 1}{x^2} = -\frac{1}{x^2}$$

$$\frac{d^2y}{dx^2} = \frac{0 - (-2x)}{x^4} = \frac{2x}{x^4} = \frac{2}{x^3}$$

[9] Find $f'(x)$ if $f(x) = \left(\frac{1}{x} + \frac{1}{x^2}\right)(3x^3 + 27)$

$$f'(x) = \left(\frac{1}{x} + \frac{1}{x^2}\right)(9x^2) + (3x^3 + 27)(-x^{-2} - 2x^{-3})$$

$$= 9x^2\left(\frac{1}{x} + \frac{1}{x^2}\right) + \left(-\frac{1}{x^2} - \frac{2}{x^3}\right)(3x^3 + 27)$$

$$= 9x + 9 - 3x - \frac{27}{x^2} - 6 - \frac{54}{x^3}$$

$$= 6x - \frac{27}{x^2} - \frac{54}{x^3} + 3$$

[10] Find $f'(x)$ if $f(x) = (x-2)(x^2+2x+4)$

$$f'(x) = (x-2)(2x+2) + (x^2+2x+4)$$

$$= 2x^2 + 2x - 4x - 4 + x^2 + 2x + 4$$

$$= \boxed{3x^2}$$

[11] Find $f'(x)$ if $f(x) = \frac{3x+4}{x^2+1}$

$$f'(x) = \frac{(x^2+1)(3) - 2x(3x+4)}{(x^2+1)^2} = \frac{3x^2+3-6x^2-8x}{(x^2+1)^2}$$

$$= \frac{-3x^2-8x+3}{(x^2+1)^2}$$

$$\therefore f'(1) = \frac{-3-2+3}{(2)^2} = \frac{-2}{4} = \left[-\frac{1}{2}\right]$$

[12] Find $f'(x)$ if $f(x) = 4\cos(x) + 2\sin(x)$

$$f'(x) = -4\sin(x) + 2\cos(x)$$

[13] Find if $f(x) = \frac{5-\cos(x)}{5+\sin(x)}$

$$f'(x) = \frac{(5+\sin(x))(\sin(x)) - (5-\cos(x))\cos(x)}{(5+\sin(x))^2}$$

$$= \frac{5\sin(x) + \sin^2(x) - 5\cos(x) + \cos^2(x)}{(5+\sin(x))^2}$$

$$= \frac{5(\sin(x) - \cos(x)) + 1}{(5+\sin(x))^2}$$

[14] $f'(x)$ if $f(x) = (x^2+1)\sec(x)$

$$f'(x) = (x^2+1)\sec(x)\tan(x) + 2x\sec(x)$$

[15] Find $f'(x)$ if $f(x) = \sin^2(x) + \cos^2(x)$

$$f(x) = 1 \rightarrow f'(x) = 0$$

[RDX]

[15] Find $F'(x)$ if $F(x) = \frac{\sec(x)}{1+\tan(x)}$

$$F'(x) = \frac{(1+\tan(x))\sec(x)\tan(x) - \sec(x)\sec(x)\tan(x)}{(1+\tan(x))^2}$$

$$= \frac{\sec(x)\tan(x) + \sec(x)\tan^2(x) - \sec^2(x)\tan(x)}{(1+\tan(x))^2}$$

[16] Find $\frac{d^2y}{dx^2} \rightarrow$ if $y = x \sin(x) + 3 \cos(x)$

$$\frac{dy}{dx} = x \cos(x) + \sin(x) + 3 \sin(x) = x \cos(x) + 4 \sin(x)$$

$$\frac{d^2y}{dx^2} = -x \sin(x) + \cos(x) + 4 \cos(x) = \boxed{-x \sin(x) + 5 \cos(x)}$$

[17] Find $\frac{d^2y}{dx^2}$ if $y = \sin(x) \cos(x)$

$$\frac{dy}{dx} = \sin(x) \cdot (-\sin(x)) + \cos(x) \cdot \cos(x)$$

$$\frac{dy}{dx} = \cos^2(x) - \sin^2(x)$$

$$\frac{d^2y}{dx^2} = 2 \cos(x) (-\sin(x)) - 2 \sin(x) \cos(x)$$

$$= -2 \sin(x) \cos(x) - 2 \sin(x) \cos(x)$$

$$\frac{d^2y}{dx^2} = \boxed{-4 \sin(x) \cos(x)}$$

[18] Find $f'(x)$ if $f(x) = \sqrt[3]{12 + \sqrt{x}}$

$$f(x) = (12 + \sqrt{x})^{1/3}$$

$$\therefore f'(x) = \frac{1}{3} (12 + \sqrt{x})^{-2/3} \cdot \frac{1}{2\sqrt{x}} = \frac{1}{6\sqrt{x}} (12 + \sqrt{x})^{-2/3}$$

[19] Find $f'(x)$ if $f(x) = 68^2 (3\sqrt{x})$

$$f(x) = (68(3\sqrt{x}))^2$$

$$\begin{aligned} f'(x) &= 2(68(3\sqrt{x})) \cdot (-\sin(3\sqrt{x})) \cdot \frac{3}{2\sqrt{x}} \\ &= -\frac{3}{\sqrt{x}} 68(3\sqrt{x}) \sin(3\sqrt{x}) \end{aligned}$$

[20] Find $f'(x)$ if $f(x) = 2 \sec^2(x^7)$

$$f(x) = 2(\sec(x^7))^2$$

$$\begin{aligned} f'(x) &= 4(\sec(x^7)) \cdot \sec(x^7) \tan(x^7) \cdot (7x^6) \\ &= 28x^6 \sec^2(x^7) \cdot \tan(x^7) \end{aligned}$$

[21] Find $f'(x)$ if $f(x) = \sqrt{3x - \sin^2(4x)}$

$$f'(x) = \frac{3 - 2\sin(4x) \cdot \cos(4x) \cdot 4}{2\sqrt{3x - \sin^2(4x)}}$$

$$= \frac{3 - 8\sin(4x)\cos(4x)}{2\sqrt{3x - \sin^2(4x)}}$$

(H.W) Find $\frac{dy}{dx}$ if

[1] $y = \frac{(3x+2)}{(x+1)}$ at $[x=1]$ -1/2

[2] $y = \sin^3(x^2+1)$ at $[x=1]$
 $\Rightarrow y'|_{x=1} = 6 \sin(2) \cos(2)$

[3] $y = e^{x^3+2x}$ at $[x=0]$ 2

[4] $y = \ln(\sin(5x)) \Rightarrow 5 \cot(5x)$

[5] $y = (x^4 + 3x^2 + 1)(x+1)^3$ at $[x=2]$
1971

[6] $y = \sin(\sec e^{x^2})$
 $y' = 2x e^{x^2} \sec(e^{x^2}) \cdot \tan(e^{x^2}) \cos(\sec e^{x^2})$

وإن كنت لا تفهم ما سأل

Date (٢٠٢٠) السبوت
د/ آلاء محمد جمال

بسم الله الرحمن الرحيم

"اشتقاق الدوال"

الدالة الأسية e^u

If $y = e^u$

$\therefore y' = u' e^u$

$e = 2.718$

عدد طبيعي

examples:

[1] $y = e^{\sqrt{x}}$

$\Rightarrow y' = \frac{1}{2\sqrt{x}} e^{\sqrt{x}}$

[2] $y = e^{\sin(x)}$

$\Rightarrow y' = \cos(x) e^{\sin(x)}$

[3] $y = e^{x \cos(x)}$

$\Rightarrow y' = (-x \sin(x) + \cos(x)) e^{x \cos(x)}$

[4] $y = \sqrt{e^{2x} + \cos(x)}$

$\Rightarrow y' = \frac{2e^{2x} - \sin(x)}{2\sqrt{e^{2x} + \cos(x)}}$

الدالة الأسية a^u

If $y = a^u$

$\Rightarrow y' = a^u \cdot u' \cdot \ln(a)$

examples:

[1] $y = 3^{5x}$

$\Rightarrow y' = 3^{5x} \cdot 5 \cdot \ln(3)$

[2] $y = \left(\frac{1}{3}\right)^{x^2}$

$\Rightarrow y = (3)^{-x^2}$

$\therefore y' = 3^{-x^2} \cdot (-2x) \cdot \ln(3)$

$$[3] \quad y = 2^{\sin(x)} \rightarrow y' = 2^{\sin(x)} \cdot \ln(2) \cdot \cos(x)$$

قواعد اللوغاريتمات

$$\bullet \ln\left(\frac{a}{b}\right) = \ln(a) - \ln(b)$$

$$\bullet \ln(ab) = \ln(a) + \ln(b)$$

$$\bullet \ln(x^n) = n \ln(x)$$

• قاعدة اللوغاريتمية $\log_b(u)$

$$\text{IF } y = \log_b(u)$$

$$y' = \frac{u'}{u \ln(b)}$$

$$\text{ex } [1] \quad y = \log_7(x^3)$$

$$y' = \frac{3x^2}{x^3 \ln(7)} = \frac{3}{x \ln(7)}$$

$$[2] \quad y = \log_8(\sqrt{x^3 + 2x})$$

$$y' = \frac{\frac{3x^2 + 2}{2\sqrt{x^3 + 2x}}}{\ln(8) \sqrt{x^3 + 2x}} = \frac{3x^2 + 2}{2 \ln(8) (x^3 + 2x)}$$

• البالة اللوغاريتمية $\ln \square$

If $y = \ln(u) \Rightarrow y' = \frac{u'}{u}$

ex 1 $y = \ln \sqrt{\sin(x)}$

$$y' = \frac{\frac{\cos(x)}{2\sqrt{\sin(x)}}}{\sqrt{\sin(x)}} = \frac{\cos(x)}{2\sin(x)} = \left(\frac{1}{2} \cot(x) \right)$$

جد $y = \ln(\sin(x))^{1/2} = \frac{1}{2} \ln(\sin(x))$

$$y' = \frac{1}{2} \frac{\cos(x)}{\sin(x)} = \frac{1}{2} \cot(x)$$

2 $y = \ln(\sin(x) \cdot \cos(x))$

$$y = \ln(\sin(x)) + \ln(\cos(x))$$

$$\therefore y' = \frac{\cos(x)}{\sin(x)} + \frac{-\sin(x)}{\cos(x)} = \cot(x) - \tan(x)$$

جد $y' = \frac{-\sin^2(x) + \cos^2(x)}{\sin(x)\cos(x)} = \frac{-\tan(x) + \cot(x)}{1}$

3 $y = \ln \sqrt[3]{4x^2 + 5x}$

$$y = \ln(4x^2 + 5x)^{1/3} = \frac{1}{3} \ln(4x^2 + 5x)$$

$$y' = \frac{1}{3} \frac{8x + 5}{4x^2 + 5x}$$

ROX

Date: دالة كبرى صفة الانسان لست (2)

Find y' for the following:

II) $y = (\sin x)^{\ln x^2}$

"The Solution"

$$\therefore y = (\sin x)^{\ln x^2}$$

$$\ln y = \ln(\sin x)^{\ln x^2} \Rightarrow \ln y = \ln x^2 \cdot \ln(\sin x)$$

$$\frac{y'}{y} = \ln x^2 \cdot \frac{\cos \sqrt{x} \cdot \frac{1}{2\sqrt{x}}}{\sin \sqrt{x}} + \frac{2x}{x^2} \ln \sin(\sqrt{x})$$

$$\frac{y'}{y} = \frac{1}{2\sqrt{x}} \ln x^2 + \frac{2}{x} \ln(\sin x)$$

$$\therefore y' = (\sin \sqrt{x})^{\ln x^2} \left[\frac{1}{2\sqrt{x}} \ln x^2 \cot(\sqrt{x}) + \frac{2}{x} \ln \sin \sqrt{x} \right]$$

Q. 9. $y = (x^3 - 2x)^{\ln x}$

"The Solution"

$$\ln y = \ln (x^3 - 2x) \ln x$$

$$\ln(y) = \ln(x) + \ln(x^3 - 2x) \quad \text{بالقائض}$$

~~$$y' = \ln(x) \cdot \frac{3x^2-2}{x^3-2x} + \ln(x^3-2x) \cdot \frac{1}{x}$$~~

$$y' = (x^3 - 2x)^{\ln x} \left[\frac{3x^2 - 2}{x^3 - 2x} \ln(x) + \frac{1}{x} \ln(x^3 - 2x) \right]$$

$$[G(x)]^X$$

بناؤں میں / م / و / والے (۱) عربی حروف کسے کہلاتے ہیں

$$\frac{(4x^3 + 2) \sin(5x)}{x^2 e^{3x}}$$

م / د. أسراء العزوي

- Find $\frac{d^2y}{dx^2}$ by implicit differentiation

1] $2x^2 - 3y^2 = 4$

$4x - 6yy' = 0$ بالاشتقاق مرة أخرى

$4 - 6(yy'' + y'y') = 0 \Rightarrow 4 - 6yy'' - 6y'^2 = 0$

$\therefore 6yy'' = 4 - 6y'^2 \Rightarrow y'' = \frac{4 - 6y'^2}{6y}$ //

2] $y + \sin(y) = x$

$y' + y' \cos(y) = 1$ بالاشتقاق مرة أخرى

$y'' + y'(-y' \sin(y)) + y' \cos(y) = 1$

$y''(1 + \cos(y)) = 1 + y'^2 \sin(y)$

$\therefore y'' = \frac{1 + y'^2 \sin(y)}{1 + \cos(y)}$ //

- Find $\frac{dy}{dx}$ if

3] $y = x^2 \log_2(3-2x)$

$y' = x^2 \cdot \frac{-2}{(3-2x)\ln(2)} + 2x \log_2(3-2x)$

$\therefore y' = \frac{-2x^2}{(3-2x)\ln(2)} + 2x \log_2(3-2x)$

$$[2] \quad y = \log_3 (\sin^2(x))$$

↓

$$y' = \frac{2 \sin(x) \cos(x)}{\sin^2(x) \ln(3)}$$

$$[3] \quad y = \ln(\cos^2(x) \sqrt{1+x^4})$$

↓

$$y = \ln(\cos^2(x)) + \ln(\sqrt{1+x^4})$$

$$y = \ln(\cos(x))^2 + \ln(\sqrt{1+x^4})$$

$$y = 2 \ln(\cos(x)) + \ln \sqrt{1+x^4}$$

$$\therefore y' = \frac{-2 \sin(x)}{\cos(x)} + \frac{4x^3}{2 \sqrt{1+x^4}}$$

$$(y' = -2 \tan(x) + \frac{2x^3}{\sqrt{1+x^4}}) \quad \#$$

$$[4] \quad y = \ln(\cos(e^x))$$

↓

$$y' = \frac{-e^x \sin(e^x)}{\cos(e^x)} = -e^x \tan(e^x)$$

$$[5] \quad y = 2^{\cos(x) + \ln(x)}$$

↓

$$y = 2^{\cos(x) + \ln(x)} = 2^{\cos(x)} \cdot 2^{\ln(x)}$$

الآن

$$y' = 2^{\cos(x)} \left(\frac{1}{x} 2^{\ln(x)} \ln(2) + 2^{\ln(x)} (-\sin(x)) 2^{\cos(x)} \ln(2) \right)$$

$$\therefore y' = \frac{1}{x} 2^{\cos(x) + \ln(x)} \ln(2) - \sin(x) \cdot 2^{\ln(x) + \cos(x)} \ln(2)$$

$$y' = 2^{\cos(x) + \ln(x)} \ln(2) \left[\frac{1}{x} - \sin(x) \right]$$

[6] $y = \frac{\sin(x) \cos(x) \tan^3(x)}{\sqrt{x}}$ دالة كسرية معقدة

الحل

$\therefore y = \frac{\sin(x) \cos(x) \tan^3(x)}{\sqrt{x}}$ أخذنا اللوغاريتم

$$\ln(y) = \ln(\sin(x) \cos(x) \tan^3(x)) - \ln(\sqrt{x})$$

$$= \ln(\sin(x)) + \ln(\cos(x)) + \ln(\tan^3(x)) - \ln(\sqrt{x})$$

$$\ln(y) = \ln(\sin x) + \ln(\cos x) + 3 \ln|\tan x| - \ln(\sqrt{x})$$

ونأخذ المشتقة

$$\frac{y'}{y} = \frac{\cos(x)}{\sin(x)} - \frac{\sin(x)}{\cos(x)} + \frac{3 \sec^2(x)}{\tan(x)} - \frac{1}{2\sqrt{x}}$$

$$\therefore y' = y \left[\cot(x) - \tan(x) - \frac{3}{\sin(x) \cos(x)} - \frac{1}{2\sqrt{x}} \right]$$

$$y' = \frac{\sin(x) \cos(x) \tan^3(x)}{\sqrt{x}} \left[\cot(x) - \tan(x) - \frac{3}{\sin(x) \cos(x)} - \frac{1}{2\sqrt{x}} \right]$$

والجواب

$$[7] y = x \sqrt[3]{1+x^2}$$

↓

$$y = x (1+x^2)^{1/3}$$

$$y' = \frac{x}{3} (2x) (1+x^2)^{-2/3} + (1+x^2)^{1/3}$$

$$\therefore y' = \frac{2x^2}{3(1+x^2)^{2/3}} + (1+x^2)^{1/3}$$

#

طريقة أخرى

$$y = x (1+x^2)^{1/3}$$

أخذنا اللوغاريتم

$$\ln(y) = \ln(x) + \frac{1}{3} \ln(1+x^2)$$

بمفاضة اللوغاريتم

$$\therefore y' = y \left[\frac{1}{x} + \frac{2x}{3(1+x^2)} \right] = x(1+x^2)^{1/3} \left[\frac{1}{x} + \frac{2x}{3(1+x^2)} \right]$$

$$\therefore y' = (1+x^2)^{1/3} + \frac{2x^2}{3(1+x^2)^{2/3}}$$

#

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# مشتقات (السؤال المتلوية العكسية)

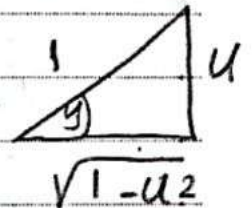
\* IF  $y = \sin^{-1}(u)$  Find  $\frac{dy}{dx}$

$\therefore y = \sin^{-1}(u)$  بالمثل في  $\sin$  للطرفين

$\sin(y) = \sin \sin^{-1}(u) \Rightarrow \sin(y) = u$   
بقياس  $u$  في

$\therefore y' \cos(y) = u'$

$\therefore y' = \frac{u'}{\cos(y)} = \frac{u'}{\sqrt{1-u^2}}$



$\therefore \frac{d}{dx} \sin^{-1}(u) = \frac{u'}{\sqrt{1-u^2}}$

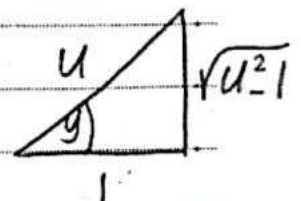
\* IF  $y = \sec^{-1}(u)$  Find  $\frac{dy}{dx}$

$\therefore y = \sec^{-1}(u)$  بالمثل في  $\sec$  للطرفين

$\sec(y) = \sec \sec^{-1}(u) \Rightarrow \sec(y) = u$   
بقياس  $u$  في

$\therefore y' \sec(u) \tan(u) = u'$

$y' = \frac{u'}{\sec(u) \tan(u)} = \frac{u'}{u \sqrt{u^2-1}}$



$\therefore \frac{d}{dx} (\sec^{-1}(u)) = \frac{u'}{u \sqrt{u^2-1}}$

y

$$\sin^{-1}(u)$$

$$\cos^{-1}(u)$$

$$\tan^{-1}(u)$$

$$\cot^{-1}(u)$$

$$\sec^{-1}(u)$$

$$\csc^{-1}(u)$$

y'

$$\frac{u'}{\sqrt{1-(u')^2}}$$

$$\frac{-u'}{\sqrt{1-(u')^2}}$$

$$\frac{1+(u')^2}{u'}$$

$$\frac{1+(u')^2}{-u'}$$

$$\frac{u'\sqrt{(u')^2-1}}{u'}$$

$$\frac{-u'\sqrt{(u')^2-1}}{u'\sqrt{(u')^2-1}}$$

\* Find  $\frac{dy}{dx}$  if

$$[1] \quad y = \sin^{-1}(3x)$$

$u = 3x$

$$\therefore \frac{d \sin^{-1}(u)}{dx} = \frac{u'}{\sqrt{1-(u')^2}}$$

$$\therefore u = 3x \Rightarrow u' = 3$$

$$\therefore y' = \frac{3}{\sqrt{1-(3x)^2}}$$

$$y' = \frac{3}{\sqrt{1-9x^2}} \quad \#$$

$$[2] \quad y = \tan^{-1}(x^3)$$

$u = x^3$

$$y' = \frac{3x^2}{1+(x^3)^2} = \frac{3x^2}{1+x^6}$$

$$[3] \quad y = \csc^{-1}(e^x)$$

$u = e^x$

$$y' = \frac{-e^x}{e^x \sqrt{(e^x)^2-1}} = \frac{-1}{\sqrt{e^{2x}-1}}$$

$$[4] \quad y = \sin^{-1}(\ln(x))$$

↓

$$y' = \frac{1/x}{\sqrt{1 - (\ln(x))^2}} = \frac{1}{x \sqrt{1 - (\ln(x))^2}}$$

~~///~~

$$[5] \quad y = x^2 \cos^{-1}\left(\frac{1}{x}\right) - \sqrt{x^2 - 1}$$

↓

$$y' = x^2 \cdot \frac{-(-\frac{1}{x^2})}{\sqrt{1 - (\frac{1}{x})^2}} + 2x \cos^{-1}\left(\frac{1}{x}\right) - \frac{2x}{2\sqrt{x^2 - 1}}$$

$$= \frac{1}{\sqrt{1 - (\frac{1}{x})^2}} + 2x \cos^{-1}\left(\frac{1}{x}\right) - \frac{x}{\sqrt{x^2 - 1}}$$

$$= \frac{x}{\sqrt{x^2 - 1}} + 2x \cos^{-1}\left(\frac{1}{x}\right) - \frac{x}{\sqrt{x^2 - 1}}$$

$$\therefore y' = 2x \cos^{-1}\left(\frac{1}{x}\right)$$

~~~~~

مشتقات الدوال الزائدية

y

y'

$$\sinh(u)$$

$$u' \cosh(u)$$

$$\cosh(u)$$

$$u' \sinh(u)$$

$$\tanh(u)$$

$$u' \operatorname{sech}^2(u)$$

$$\operatorname{sech}(u)$$

$$-u' \operatorname{sech}(u) \tanh(u)$$

$$\operatorname{csch}(u)$$

$$-u' \operatorname{csch}(u) \coth(u)$$

$$\coth(u)$$

$$-u' \operatorname{csch}^2(u)$$

Find $\frac{dy}{dx}$ if

1) $y = \operatorname{sech}(3x^2+1) + \sinh \sqrt[3]{x^5}$ $\rightarrow x^{5/3}$

$$y' = -6x \operatorname{sech}(3x^2+1) \tanh(3x^2+1) + \frac{5}{3} x^{2/3} \cosh(\sqrt[3]{x^5})$$

2) $\tanh(e^y) + \cosh(\coth(x^2+1)) = e^{3x}$

$$y' e^y \operatorname{sech}^2(e^y) - \operatorname{csch}^2(x^2+1) (2x) \sinh(\coth(x^2+1)) = 3e^{3x}$$

$$\therefore y' e^y \operatorname{sech}^2(e^y) = 3e^{3x} + 2x \operatorname{csch}^2(x^2+1) \sinh(\coth(x^2+1))$$

$$\therefore y' = \frac{1}{e^y \operatorname{sech}^2(e^y)} [3e^{3x} + 2x \operatorname{csch}^2(x^2+1) \sinh(\coth(x^2+1))]$$

الاجابة

"تفاضل الدوال الزائدية العكسية"

IF $y = \sinh^{-1}(u)$ find $\frac{dy}{dx}$?

الحل

$\therefore y = \sinh^{-1}(u)$ (بغير الطرف في $\sinh$)

$\sinh(y) = \sinh \sinh^{-1}(u) \Rightarrow \sinh(y) = u \rightarrow (X)$
تفاضل الطرفين بالنسبة لـ x

$$\therefore y' \cosh(y) = u' \Rightarrow \therefore y' = \frac{u'}{\cosh(y)}$$

$$\therefore \cosh^2(y) - \sinh^2(y) = 1 \Rightarrow \cosh(y) = \sqrt{1 + \sinh^2 y}$$

$$\therefore y' = \frac{u'}{\sqrt{1 + \sinh^2(y)}} = \frac{u'}{\sqrt{1 + u^2}}$$

$$\therefore \left(\frac{dy}{dx} = \frac{u'}{\sqrt{1 + u^2}} \right)$$

"مشتقات الدوال الزائدية العكسية"

y
 $\sinh^{-1}(u)$

$\cosh^{-1}(u)$

$\tanh^{-1}(u), \coth^{-1}(u)$

$\operatorname{sech}^{-1}(u)$

$\operatorname{csch}^{-1}(u)$

y'

$\frac{u'}{\sqrt{u^2 + 1}}$

$\frac{u'}{\sqrt{u^2 - 1}}$

$\frac{u'}{1 - u^2}$

$\frac{-u'}{u\sqrt{1 - u^2}}$

$\frac{-u'}{u\sqrt{1 + u^2}}$

$\frac{-u'}{u\sqrt{1 + u^2}}$

$\frac{-u'}{u\sqrt{1 + u^2}}$

[1] If $y = \sinh^{-1}(e^{x^2+1})$ find y' ?

$$y' = \frac{2xe^{x^2}}{\sqrt{(e^{x^2+1})^2 + 1}} = \frac{2xe^{x^2}}{\sqrt{e^{2x^2+2} + 1}} \quad \#$$

[2] If $y = \tanh^{-1}(3x)$ find y'

$$y' = \frac{3}{1 - 9x^2} \quad \#$$

"Taylor and Maclorain series"

متسلسلة تايلور وماكلورين :

مفكوك تايلور :-

$$P(x) = P(x_0) + \frac{P'(x_0)}{1!} (x-x_0) + \frac{P''(x_0)}{2!} (x-x_0)^2 + \frac{P'''(x_0)}{3!} (x-x_0)^3 + \dots$$

مفكوك ماكلورين :- (at $x_0=0$)

$$P(x) = P(0) + \frac{P'(0)}{1!} x + \frac{P''(0)}{2!} x^2 + \frac{P'''(0)}{3!} x^3 + \dots$$

* رعي مفكوك ماكلورين حالة خاصة من مفكوك تايلور
عندما $x_0=0$

ex Find taylor series for $P(x) = \ln(x)$ with $x_0=1$?

$$P(x) = P(x_0) + \frac{P'(x_0)}{1!} (x-x_0) + \frac{P''(x_0)}{2!} (x-x_0)^2 + \dots$$

$$P(x_0) = \ln(x_0) = \ln(1) = 0$$

$$P'(x) = \frac{1}{x} \Rightarrow P'(1) = 1$$

$$P''(x) = \frac{-1}{x^2} \Rightarrow P''(1) = -1$$

$$P'''(x) = \frac{2}{x^3} \Rightarrow P'''(1) = 2$$

$$\therefore P(x) = (x-1) + \frac{1}{2} (-1) (x-1)^2 + \frac{2}{3!} (1) (x-1)^3 + \dots$$

النتيجة

$$\ln(x) = (x-1) - \frac{1}{2} (x-1)^2 + \frac{2}{6} (x-1)^3 + \dots$$

* Find Maclorain series For $f(x) = \sin(x)$

$x \ll 1$

$$f(x) = f(0) + \frac{f'(0)}{1!}x + \frac{f''(0)}{2!}x^2 + \frac{f'''(0)}{3!}x^3 + \dots$$

$$f(0) = 0, \quad f'(0) = 1, \quad f''(0) = 0$$

$$\therefore \sin(x) = x + 0 - \frac{1}{3!}x^3 + 0 + \dots$$

$$\therefore \sin(x) = x - \frac{1}{6}x^3 + \frac{1}{5!}x^5 + \dots$$

P

« بسم الله الرحمن الرحيم »

السبب الرابع

التكامل باستخدام الكسور الجزئية :-

تفاضل وتكامل 2

م / أسراء العزوني

في كثير من الأحيان لا يمكن تطبيق طريقة عكس المشتقة مباشرة وذلك بسبب عدم معرفتنا لهذه المشتقة ، أو بسبب كون التكامل معطى في صورة يصعب معها استخدام مفهوم عكس المشتقة ، ولذلك نلجأ لاستخدام طرق أخرى للتكامل وهي [١] التكامل بالتكويين ، [٢] التكامل بالكسور الجزئية [٣] التكامل بالتجزئ .

إذا كانت الدالة $P(X)$ عبارة

$$P(X) = \frac{X}{X-1} + \frac{3}{X} \xrightarrow{\text{بتوضيع المقام}} \frac{X^2 + 3X - 3}{X(X-1)}$$

فعلينا أن نضع مجموعة الكسور تحولت لكسر واحد بعد عملية تقوية المقامات
موضوعنا التفارعة كيفية تحويل كسر واحد لعدد من الكسور وذلك باستخدام
عملية الكسور الجزئية .
ولذلك نقرر نقول بعملية الكسور الجزئية عملية عكسية لتوضيع المقامات
والعكس صحيح .

• الشروط الواجب توافرها لتطبيق عملية الكسور الجزئية .
[١] درجة البسط < درجة المقام . [٢] المقام لا يحوي على جذور
يوجب 4 سالبة و 4 سالبة .

[١] جميع عوامل المقام من الدرجة الأولى حقيقية غير مكررة .

[٢] بعض عوامل المقام من الدرجة الأولى حقيقية مكررة .

[٣] بعض عوامل المقام دالة تربيعية غير قابلة للتفكيك .

[٤] المقام يحوي على عوامل مكررة من الدرجة الثانية لتحلل العوامل الحقيقية

الحالة الأولى :- جميع عوامل المقام من الدرجة الأولى حقيقة غير مكررة ،

(2) توجد التكامل (النوع II) $\int \frac{3}{(x+3)(2x^2+3x+1)} dx$

بما أن درجة المقام > درجة البسط $\rightarrow$ ^{الحل} بتفادي الكسور الجزئية.

$$\int \frac{3}{(x+3)(2x^2+3x+1)} dx = \int \frac{3}{(x+3)(2x+1)(x+1)} dx$$

بتفادي الكسور الجزئية

$$\frac{3}{(x+3)(2x+1)(x+1)} = \frac{A}{(x+3)} + \frac{B}{(2x+1)} + \frac{C}{x+1}$$

$$3 = A(2x+1)(x+1) + B(x+3)(x+1) + C(x+3)(2x+1)$$

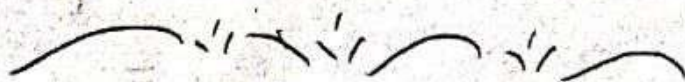
$$\text{at } \boxed{x = -1} \Rightarrow 3 = -2C \Rightarrow C = \frac{-3}{2}$$

$$\text{at } \boxed{x = -3} \Rightarrow 3 = 2A \Rightarrow A = \frac{3}{2}$$

$$\text{at } \boxed{x = -\frac{1}{2}} \Rightarrow 3 = \frac{5}{4}B \Rightarrow B = \frac{12}{5}$$

$$\therefore \int \frac{3}{(x+3)(2x+1)(x+1)} dx = \int \frac{3/2}{(x+3)} dx + \int \frac{12/5}{(2x+1)} dx + \int \frac{-3/2}{(x+1)} dx$$

$$= \frac{3}{2} \ln|x+3| + \frac{6}{5} \ln|2x+1| - \frac{3}{2} \ln|x+1|$$



(مثال)

تقسيم الكسور الجزئية

$$\frac{2+x}{x(3x-1)(x+2)}$$

الحل

$$\frac{2+x}{x(3x-1)(x+2)} = \frac{A}{x} + \frac{B}{(3x-1)} + \frac{C}{(x+2)}$$

$$2+x = A(x+2)(3x-1) + B(x)(x+2) + C(x)(3x-1)$$

تعيين قيم x $\rightarrow$ $x=0$, $x=\frac{1}{3}$, $x=-2$

at $x=0$ $2 = -2A \Rightarrow A = -1$

at $x=-2$ $0 = 14C \Rightarrow C = 0$

at $x=\frac{1}{3}$ $\frac{7}{3} = \frac{7}{9}B \Rightarrow B = 3$

$$\therefore \frac{2+x}{x(3x-1)(x+2)} = \frac{-1}{x} + \frac{3}{(3x-1)} + \frac{0}{(x+2)}$$

$$\frac{2+x}{x(3x-1)(x+2)} = \frac{-1}{x} + \frac{3}{(3x-1)}$$

~~///~~

اللهم إني أَسْأَلُكَ عِلْمًا نَافِعًا وَكَلِمًا مَقْبُولًا

د/عبدالله محمد زكي

α

بعض عوامل المقام من الدرجة الأول ولكننا مكررة

الحالة الثالثة

$$\frac{x^2 + x + 2}{(x-1)^3 (x-2)}$$

$$\frac{x^2 + x + 2}{(x-1)^3 (x-2)} = \frac{A}{(x-1)^3} + \frac{B}{(x-1)^2} + \frac{C}{(x-1)} + \frac{D}{(x-2)}$$

عوامل المقام من الدرجة الأول

$$x^2 + x + 2 = A(x-2) + B(x-1)(x-2) + C(x-1)^2(x-2) + D(x-1)^3$$

أضمار المقام $\Leftarrow \boxed{x=2}, \boxed{x=1}$

at $\boxed{x=1}$ $4 = -A \Rightarrow \boxed{A = -4}$

at $\boxed{x=2}$ $\boxed{8 = D}$

at $\boxed{x=0}$ $2 = -2A + 2B - 2C - D$
 $2B - 2C = 2 \quad (1)$

at $\boxed{x=-1}$ $2 = -3A + 6B - 12C - 8 \quad (2)$

$54 = 6B - 12C \quad (2)$

نحل (1) و (2) معاً

$\boxed{B = -7}, \boxed{C = -8}$

$$\therefore \frac{x^2 + x + 2}{(x-1)^3 (x-2)} = \frac{-4}{(x-1)^3} - \frac{7}{(x-1)^2} - \frac{8}{(x-1)} + \frac{8}{(x-2)}$$

~~///~~

[4]

الخطوة التالية : بعثت عوامل المقام حقيقة مكررة

$$[3] \int \frac{x^2+1}{(x-1)^2(x+1)^2} dx$$

الحل
بالتفاد الكسور الجزئية

$$\frac{(x^2+1)}{(x-1)^2(x+1)^2} = \frac{A}{(x-1)^2} + \frac{B}{(x-1)} + \frac{C}{(x+1)^2} + \frac{D}{(x+1)}$$

$$x^2+1 = A(x+1)^2 + B(x+1)^2(x-1) + C(x-1)^2 + D(x+1)(x-1)^2$$

$$\text{at } [x=-1] \Rightarrow 2 = 4C \Rightarrow C = \frac{1}{2}$$

$$[x=1] \Rightarrow 2 = 4A \Rightarrow A = \frac{1}{2}$$

$$[x=0] \Rightarrow 1 = A - B + C + D \Rightarrow B + D = 0 \quad (1)$$

$$[x=2] \Rightarrow 5 = 9A + 9B + C + 3D \Rightarrow 9B + 3D = 0 \quad (2)$$

$$[B=D=0] \quad \text{كل 1 و 2 صفاً نجد 0}$$

$$\therefore \int \frac{x^2+1}{(x-1)^2(x+1)^2} dx = \int \frac{1/2}{(x-1)^2} dx + \int \frac{1/2}{(x+1)^2} dx$$

$$= \frac{1}{2} \int (x-1)^{-2} dx + \frac{1}{2} \int (x+1)^{-2} dx$$

$$= \frac{1}{2} \left[-(x-1)^{-1} + (x+1)^{-1} \right] + C$$

$$= \frac{1}{2} \left[\frac{1}{x-1} + \frac{1}{x+1} \right] + C$$

حالة الثالثة

« بقدر كوامل »
 « مقام دالة كويسية غير قابله للتكليل »
 « لاحظ: اذا كان المقام من الدرجة الثانية جاز البسط من الدرجة الأولى
 (AX+B)

« تخضع الكسور الجزئية »

$$\frac{2}{(x-1)(x^2+x-4)}$$

الحل
 درجة المقام < درجة البسط

$$\frac{2}{(x-1)(x^2+x-4)} = \frac{A}{(x-1)} + \frac{Bx+C}{x^2+x-4}$$

$$2 = A(x^2+x-4) + (Bx+C)(x-1)$$

at $x=1$ $2 = -2A \Rightarrow A = -1$

at $x=0$ $2 = -4A - C \Rightarrow C = -2$

at $x=-1$ $2 = -4A + 2B - 2C \Rightarrow -6 = 2B$
 $\therefore B = -3$

$$\therefore \frac{2}{(x-1)(x^2+x-4)} = \frac{-1}{(x-1)} - \frac{3x+2}{(x^2+x-4)}$$

« كن عرفت التغيير
 الذي تريد أن تراه
 في العالم »

د/ السراء العزوني

$$\frac{x^3 - x + 4}{(x-1)(x+1)(x^2+1)}$$

قسم الكسور الجزئية

$$(x-1)(x+1)(x^2+1)$$

$$\frac{x^3 - x + 4}{(x-1)(x+1)(x^2+1)} = \frac{A}{(x-1)} + \frac{B}{(x+1)} + \frac{Cx+D}{(x^2+1)}$$

$$x^3 - x + 4 = A(x+1)(x^2+1) + B(x-1)(x^2+1) + (Cx+D)(x-1)(x+1)$$

at $\boxed{x=1}$

$$4 = 4A \Rightarrow \boxed{A=1}$$

at $\boxed{x=-1}$

$$2 = -4B \Rightarrow \boxed{B = -\frac{1}{2}}$$

at $\boxed{x=0}$

$$4 = A - B - D \Rightarrow \boxed{D = -\frac{5}{2}}$$

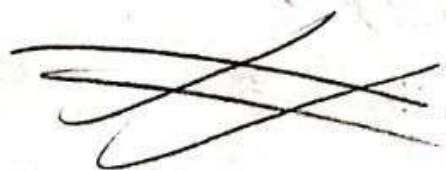
$\boxed{x=2}$

$$10 = 15A + 5B + 6C + 3D$$

$$10 = 5 + 6C \Rightarrow 5 = 6C$$

$$\therefore \boxed{C = \frac{5}{6}}$$

$$\therefore \frac{x^3 - x + 4}{(x-1)(x+1)(x^2+1)} = \frac{1}{x-1} + \frac{-1/2}{(x+1)} + \frac{5/6 - 5/2}{(x^2+1)}$$



المسألة الثانية

$$\int \frac{dx}{(x-1)(x^2+x+4)}$$

الحل

بأنضام الكسور الجزئية

$$\frac{1}{(x-1)(x^2+x+4)} = \frac{A}{x-1} + \frac{Bx+C}{x^2+x+4}$$

$$1 = A(x^2+x+4) + (Bx+C)(x-1)$$

at $x=1 \Rightarrow 1 = 6A + 0 \Rightarrow A = \frac{1}{6}$

at $x=0 \Rightarrow 1 = 4A - C \Rightarrow C = 4A - 1$

$$\therefore C = \frac{1}{3}$$

at $x=-1 \Rightarrow 1 = 4A + 2B - 2C$

$$2B = 1 + 2C - 4A \Rightarrow B = \frac{1}{6}$$

$$\therefore \int \frac{1}{(x-1)(x^2+x+4)} dx = \int \frac{1/6}{x-1} dx + \int \frac{-1/6x - 1/3}{x^2+x+4}$$

$$= \frac{1}{6} \ln|x-1| - \frac{1}{18} \int \frac{3x+6}{x^2+x+4} dx$$

$$= \frac{1}{6} \ln|x-1| - \frac{3}{18} \int \frac{x+2}{x^2+x+4} dx$$

$$= \frac{1}{6} \ln|x-1| - \frac{3}{36} \ln|x^2+x+4| - \frac{3}{36} \int \frac{3}{x^2+x+4} dx$$

$$= \frac{1}{6} \ln|x-1| - \frac{1}{12} \ln|x^2+x+4| - \frac{1}{4} \int \frac{1}{(x+1/2)^2 + 15/4} dx$$

$$\left\{ \begin{array}{l} \frac{1}{2} \int \frac{2x+4}{x^2+x+4} dx \\ \frac{1}{2} \int \frac{2x+1}{x^2+x+4} + \frac{3}{x^2+x+4} dx \end{array} \right.$$

وكما لمربع

(2) $\int \frac{5x+3}{x^3-2x^2-3x} dx$

الحل
ملاحظة: درجة المقام أكبر من درجة البسط $\Rightarrow$ ببقاء الكسور الجزئية

$$\frac{5x+3}{x^3-2x^2-3x} = \frac{5x+3}{x(x^2-2x-3)} = \frac{5x+3}{x(x-3)(x+1)}$$

$$\frac{5x+3}{x(x-3)(x+1)} = \frac{A}{x} + \frac{B}{(x-3)} + \frac{C}{(x+1)}$$

$$5x+3 = A(x-3)(x+1) + B(x)(x+1) + C(x)(x-3)$$

at $\boxed{x=3} \Rightarrow 18 = 12B \Rightarrow \therefore B = \frac{3}{2}$

$\boxed{x=-1} \Rightarrow -2 = 4C \Rightarrow \therefore C = -\frac{1}{2}$

$x=0 \Rightarrow 3 = -3A \Rightarrow \therefore A = -1$

$$\begin{aligned} \therefore \int \frac{5x+3}{x^3-2x^2-3x} dx &= \int \frac{-1}{x} dx + \int \frac{3/2}{(x-3)} dx + \int \frac{-1/2}{x+1} dx \\ &= -\ln|x| + \frac{3}{2} \ln|x-3| - \frac{1}{2} \ln|x+1| + C \end{aligned}$$

كلها الباقى
السيكست

$$\int \frac{3x+13}{x^2+4x+3} dx = \int \frac{3x+13}{(x+1)(x+3)} dx$$

$$\Rightarrow \int \frac{5}{x+1} dx + \int \frac{-2}{x+3} dx = 5 \ln|x+1| - 2 \ln|x+3| + C$$

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x اذا كانت المخرج غير معروف 0/0 او ∞/∞ كمية غير معينة

$$\lim_{x \rightarrow a} \frac{f(x)}{g(x)} = \lim_{x \rightarrow a} \frac{f'(x)}{g'(x)}$$

ex $\lim_{x \rightarrow \pi/2} \frac{\tan(x)}{\tan(3x)}$

$$\lim_{x \rightarrow \pi/2} \frac{\tan(x)}{\tan(3x)} = \frac{\infty}{\infty}$$

الحل

استخدام قاعدة لوبيتال

$$\lim_{x \rightarrow \pi/2} \frac{\sin(x) \cos(3x)}{\cos(x) \sin(3x)} = \lim_{x \rightarrow \pi/2} \frac{-\sin(x) \sin(3x) + \cos(x) \cos(3x)}{3 \cos(x) \cos(3x) - \sin(x) \sin(3x)}$$

$$= \boxed{3}$$

$$\lim_{x \rightarrow \infty} (x \sin(\frac{a}{x}))$$

الحل

$$\frac{1}{\infty} = 0$$

وضعناها في هذه الصورة
عشان نفهم لوبيتال

$$\lim_{x \rightarrow \infty} \frac{\sin(a/x)}{1/x} = \frac{\sin(a/\infty)}{1/\infty} = \frac{0}{0}$$

استخدام قاعدة لوبيتال

$$\lim_{x \rightarrow \infty} \frac{-a/x^2 \cos(a/x)}{-1/x^2} = \lim_{x \rightarrow \infty} (a \cos(\frac{a}{x})) = a \cos(\frac{a}{\infty}) = a \cos(0) = \boxed{a}$$

$$\frac{d}{dx} \sin^{-1}(x) = \frac{1}{\sqrt{1-x^2}}$$

$$\lim_{x \rightarrow 0} \frac{x - \sin^{-1}(x)}{-\sin^2(x)} = (1-x^2)^{-1/2}$$

الحل

$$\lim_{x \rightarrow 0} \frac{x - \sin^{-1}(x)}{-\sin^2(x)} = \frac{0}{0}$$

بتطبيق قاعدة لوبيتال

$$\lim_{x \rightarrow 0} \frac{1 - \frac{1}{\sqrt{1-x^2}}}{-2 \sin(x) \cos(x)} = \frac{0}{0}$$

بتطبيق قاعدة لوبيتال مرة أخرى

$$\lim_{x \rightarrow 0} \frac{(1-2x)(\frac{1}{2})(1-x^2)^{-3/2}}{2 \sin^2(x) - 2 \cos^2(x)} = \frac{0}{-2} = \boxed{0}$$

$$\lim_{x \rightarrow 0} \frac{\ln(x)}{\csc(x)}$$

الحل

$$\lim_{x \rightarrow 0} \frac{\ln(x)}{\csc(x)} = \frac{\infty}{\infty}$$

بتطبيق قاعدة لوبيتال

$$\lim_{x \rightarrow 0} \frac{1/x}{-\csc(x) \cot(x)} = \frac{\infty}{\infty}$$

بتطبيق قاعدة لوبيتال مرة أخرى

$$\lim_{x \rightarrow 0} \frac{-\sin^2(x)}{x \cot(x) - x \sin(x) + 1} = \frac{-2 \sin(x) \cos(x)}{-x \sin(x) + 1} = \frac{0}{1} = \boxed{0}$$

$$-1 + 1 - 0$$

1 1

$$\lim_{x \rightarrow \pi/2} (\sec(x) - \tan(x))$$

↓

$$\lim_{x \rightarrow \pi/2} \left(\frac{1}{\cos(x)} - \frac{\sin(x)}{\cos(x)} \right) = \lim_{x \rightarrow \pi/2} \frac{\cos(x) - \sin(x)\cos(x)}{\cos^2(x)}$$

$$= \frac{0}{0}$$

استخدام قاعدة لوبيتال

$$\lim_{x \rightarrow \pi/2} \frac{-\sin(x) + \sin^2(x) - \cos^2(x)}{-2\cos(x)\sin(x)} = \frac{-1 + 1 - 0}{0} = \frac{0}{0}$$

استخدام لوبيتال مرة أخرى

$$\lim_{x \rightarrow \pi/2} \frac{-\cos(x) + 2\sin(x)\cos(x) + 2\cos(x)\sin(x)}{-2(\cos^2(x) - \sin^2(x))} = \frac{0 + 0 + 0}{-2} = \frac{0}{-2} = 0$$

القياسات /

الدالة تكون متصلة عند النقطة $x=a$ ، إذا لو كفت :

(1) الدالة موجودة / معرفة عند $x=a$ $\leftarrow f(a)$

(2) نهاية الدالة موجودة عند ما $x \rightarrow a$

$$f(a) = \lim_{x \rightarrow a} f(x)$$

ex

$$f(x) = \frac{x^2 - 1}{x - 1}$$

نبحث في اتصال الدالة عند $[x=1]$

الحل

(1) $f(1) = \frac{1-1}{1-1} = \frac{0}{0}$ الدالة غير موجودة عند $x=1$

(2) $\lim_{x \rightarrow 1} \frac{x^2 - 1}{x - 1} = 0$

بالاستخدام لوبيتال

$\lim_{x \rightarrow 1} \frac{2x}{1} = [2]$

الدالة هنا فيا موجودة وتساوي 2

(3) $f(1) \neq \lim_{x \rightarrow 1} \frac{x^2 - 1}{x - 1}$

الدالة غير متصلة .

f

ملحوظة هامة / ممكن أن تكون الدالة ليست فيا وفيما عند ما نقطة ولكن ممكن أن يكون فيا نهاية عند نفس النقطة .

$$ex = P(x) = \begin{cases} \frac{x^3 - 8}{x - 2} & , x \neq 2 \\ 12 & , x = 2 \end{cases}$$

أثبت أن الدالة متصلة عند $x=2$
 $\Downarrow$

[1] $P(2) = [12] \Rightarrow$ الدالة معرفة عند $x=2$

[2] $\lim_{x \rightarrow 2} \frac{x^3 - 8}{x - 2} = \frac{0}{0}$ بصيغة فاما لوبيتال

$$\lim_{x \rightarrow 2} \frac{3x^2}{1} = [12]$$

[3] $P(2) = \lim_{x \rightarrow 2} \frac{x^3 - 8}{x - 2}$

~~الخط~~ الدالة متصلة عند $x=2$

~~~~~

Exercise Set (2.3)

الحوال الأسيّة

Table (4): Exponential Function

$$\Rightarrow \frac{d}{dx} [e^{F(x)}] = e^{F(x)} \cdot F'(x)$$

$$\Rightarrow \frac{d}{dx} [a^{F(x)}] = a^{F(x)} \cdot F'(x) \cdot \ln(a) \quad a = \text{Constant}$$

$$\Rightarrow \ln(e) = 1$$

الحوال المثلثيّة العكسيّة

Table (5): Inverse Trigonometric

$$1 - \frac{d}{dx} [\sin^{-1}(u)] = \frac{1}{\sqrt{1-u^2}} \cdot du$$

$$\frac{d}{dx} [\cos^{-1}(u)] = \frac{-1}{\sqrt{1-u^2}} \cdot du$$

$$2 - \frac{d}{dx} [\tan^{-1}(u)] = \frac{1}{1+u^2} \cdot du$$

$$\frac{d}{dx} [\cot^{-1}(u)] = \frac{-1}{1+u^2} \cdot du$$

$$3 - \frac{d}{dx} [\sec^{-1}(u)] = \frac{1}{u\sqrt{u^2-1}} \cdot du$$

$$\frac{d}{dx} [\csc^{-1}(u)] = \frac{-1}{u\sqrt{u^2-1}} \cdot du$$

$$3) y = (x^3 - 5)(2x + 3)$$

عنى لمرتبتين

$$(1) y' = (3x^2)(2x + 3) + (2)(x^3 - 5)$$

$$y' = (3x^2)(2x + 3) + (2x^3 - 10)$$

$$y'' = 6x(2x + 3) + 2(3x^2) + 6x^2$$

$$y'' = 12x^2 + 18x + 6x^2 + 6x^2$$

$$y'' = 24x^2 + 18x \neq$$

$$(2) y = (x^3 - 5)(2x + 3) = 2x^4 + 3x^3 - 10x - 15$$

$$y' = 8x^3 + 9x^2 - 10$$

$$y'' = 24x^2 + 18x \neq$$

(القاعدة)

$$\frac{d}{dx} [g(x) \cdot f(x)] =$$

$$g'(x)f(x) + f'(x)g(x)$$

مشتقة الناتج = مشتقة الأولى × الثانية + الأولى × مشتقة الثانية

⊕ مشتقة الدالة الأولى × الدالة الثانية + مشتقة الدالة الثانية × الدالة الأولى

← هنا ضربت القوسين الأول ثم مشتقتهم وللازم الثانيين راجعوا نفس الناتج

### Exercises Set (2.4)

$$1- f(x) = (x+1)(x^2 - x + 1)$$

(a) Multiplying and then diff →

أضرب ثم فاخذ

$$f(x) = (x+1)(x^2 - x + 1)$$

$$f(x) = x^3 - x^2 + x + x^2 - x + 1$$

$$f(x) = x^3 + 1$$

$$f'(x) = 3x^2 \rightarrow ①$$

(b) using the Product rule →

استخدم قاعدة الضرب

$$f(x) = (x+1)(x^2 - x + 1)$$

$$f'(x) = (1)(x^2 - x + 1) + (2x - 1)(x + 1) \rightarrow \text{بشكل كل ما}$$

$$f'(x) = (x^2 - x + 1) + (2x^2 + 2x - x - 1)$$

$$f'(x) = 3x^2 \rightarrow ②$$

(القاعدة)

$$(f \cdot g)' = f'g + g'f$$

$$② = ① \therefore$$

$$6- F(x) = \sin^2 x + \cos^2 x$$

$$F(x) = 1$$

$$\sin^2 x + \cos^2 x = 1$$

$$F'(x) = \text{Zero}$$

7- Find  $\frac{dy}{dx}$  if:-

$$1- y = x \cdot \sin x - 3 \cos x$$

$$y' = \sin x + x \cos x + 3 \sin x$$

$$y' = 4 \sin x + x \cos x$$

$$y'' = 4 \cos x + \cos x - x \sin x$$

$$y'' = 5 \cos x - x \sin x$$

$$2- y = \sin x \cos x$$

$$y' = \cos x \cdot (\cos x) - \sin x (\sin x)$$

$$y' = \cos^2 x - \sin^2 x$$

$$y'' = 2 \cos x (\sin x) - 2 \sin x (\cos x)$$

$$y'' = -2 [\cos x \sin x + \sin x \cos x]$$

$$y'' = -2 [y + y] = -4y$$

$$5- y = \sqrt{3x - \sin^2(4x)}$$

$$y' = \frac{1}{2\sqrt{3x - \sin^2(4x)}} \cdot (3 - 2 \sin(4x) \cdot \cos(4x) \cdot (4))$$

$$\cos(4x) \cdot (4)$$

$$y' = \frac{[3 - 8 \sin(4x) \cos(4x)]}{2\sqrt{3x - \sin^2(4x)}}$$

## Exercise (2.6)

Find  $y'$  if

$$1- y = (3x^2 + 2x - 1)^6$$

$$y' = 6(3x^2 + 2x - 1)^5 \cdot (6x + 2)$$

$$\text{Let } u = 3x^2 + 2x - 1 \therefore du = 6x + 2$$

$$\therefore y = (u)^6 \rightarrow y' = 6u^5 \cdot du$$

$$y' = 6[3x^2 + 2x - 1]^5 \cdot (6x + 2)$$

$$2- y = \sqrt[3]{x} + 12$$

$$y = (\sqrt{x} + 12)^{\frac{1}{3}}$$

$$y' = \frac{1}{3} (\sqrt{x} + 12)^{-\frac{2}{3}} \cdot \frac{1}{2\sqrt{x}}$$

$$3- y = \cos^2(3\sqrt{x})$$

مشتقة لزاوية

$$y' = -2 \cos(3\sqrt{x}) \cdot \sin(3\sqrt{x}) \cdot \frac{3}{2\sqrt{x}}$$

$$4- y = 2 \sec^2(x^7)$$

$$y' = (2)(2) \sec(x^7) \cdot \sec(x^7) \tan(x^7) \cdot (7x^6)$$

$$y' = 4 \sec(x^7) \cdot \sec(x^7) \tan(x^7) \cdot (7x^6)$$

$$y' = 4 \sec^2(x^7) \tan(x^7) \cdot (7x^6)$$

# Integration

## Table (1) - Integration Formulas

التكامل الجبري

$$1- \int dx = x + c$$

$$2- \int x^r dx = \frac{x^{r+1}}{r+1} + c$$

$$3- \int \frac{1}{x} dx = \ln|x| + c$$

$$4- \int e^x dx = e^x + c$$

$$5- \int b^x dx = \frac{b^x}{\ln b} + c$$

$$6- \int F(x)^n F'(x) dx = \frac{F(x)^{n+1}}{n+1} + c$$

$$7- \int \frac{F'(x)}{F(x)} dx = \ln|F(x)| + c$$

التكامل التفاضلي

$$1- \int \frac{1}{1+x^2} dx = \tan^{-1}x + c \quad \text{or} \quad -\cot^{-1}x + c$$

$$\int \frac{1}{\sqrt{1-x^2}} dx = \sin^{-1}x + c \quad \text{or} \quad -\cos^{-1}x + c$$

$$\int \frac{1}{x\sqrt{1-x^2}} dx = \sec^{-1}x + c \quad \text{or} \quad -\csc^{-1}x + c$$

$$1- \int \sin x dx = -\cos x + c$$

$$2- \int \cos x dx = \sin x + c$$

$$3- \int \tan x dx = -\ln|\cos x| + c = \ln|\sec x| + c$$

$$4- \int \sec^2 x dx = \tan x + c$$

$$5- \int \csc^2 x dx = -\cot x + c$$

$$6- \int \sec x \tan x = \sec x + c$$

$$7- \int \csc x \cot x = -\csc x + c$$

$$1- \int \sin^n x dx = -\frac{1}{n} \sin^{n-1} x \cos x + \frac{n-1}{n} \int \sin^{n-2} x dx$$

$$2- \int \cos^n x dx = \frac{1}{n} \cos^{n-1} x \sin x + \frac{n-1}{n} \int \cos^{n-2} x dx$$

$$3- [\sin ax \cos bx] = \frac{1}{2} [\sin(a+b)x + \sin(a-b)x]$$

$$4- [\cos ax \cos bx] = \frac{1}{2} [\cos(a+b)x + \cos(a-b)x]$$

$$5- [\sin ax \sin bx] = \frac{1}{2} [\cos(a-b)x - \cos(a+b)x]$$

$$\sin^2 x = \frac{1}{2} (1 - \cos 2x)$$

$$\cos^2 x = \frac{1}{2} (1 + \cos 2x)$$

(F)

# Exercises Set (3-3)

⇒ Find  $\frac{dy}{dx}$  if:-

1-  $y = e^{x \tan x}$

$y' = e^{x \tan x} \cdot (\tan x + x \sec^2 x)$

3-  $y = \pi^{\sin x}$

→  $y' = \pi^{\sin x} \cdot \cos x \cdot \ln \pi$

2-  $y = \ln(\cos e^x)$

$y' = \frac{-\sin(e^x) \cdot e^x}{\cos(e^x)} = -e^x \tan(e^x)$

القاعدة  
 $\frac{d}{dx} (a^{f(x)}) = a^{f(x)} \cdot f'(x) \cdot \ln a$

Find  $\frac{dy}{dx}$  using the Method of logarithmic diff

1-  $y = (x^3 - 2x)^{\ln x}$

أخذ لو اللطرين

$\ln y = \ln (x^3 - 2x)^{\ln x}$

$\ln y = \ln x \cdot \ln (x^3 - 2x)$

$\frac{1}{y} y' = \frac{1}{x} \ln (x^3 - 2x) + \ln x \cdot \frac{3x^2 - 2}{x^3 - 2x}$

$y' = y \left[ \frac{\ln (x^3 - 2x)}{x} + \frac{\ln x (3x^2 - 2)}{x^3 - 2x} \right]$

$y' = (x^3 - 2x)^{\ln x} \left[ \frac{\ln (x^3 - 2x)}{x} + \frac{\ln x (3x^2 - 2)}{x^3 - 2x} \right] \neq$

2-  $y = x^{\sin x}$

أخذ لو اللطرين

$\ln y = \ln x^{\sin x}$

$\ln y = \sin x \ln x$

$\frac{1}{y} \cdot y' = \cos x \ln x + \frac{\sin x}{x}$

$y' = y \left[ \cos x \ln x + \frac{\sin x}{x} \right]$

$y' = x^{\sin x} \left[ \cos x \ln x + \frac{\sin x}{x} \right] \neq$

$$6-y = x^3 \sin^2(5x)$$

$$y' = 3x^2 [\sin^2(5x)] + [x^3] (2 \sin(5x) \cdot \cos(5x) \cdot 5)$$

$$y' = 3x^2 [\sin^2(5x)] + [x^3] [10 \sin(5x) \cos(5x)]$$

$$7-y = \sqrt{x} \tan^3(\sqrt{x})$$

$$y' = \frac{1}{2\sqrt{x}} [\tan^3(\sqrt{x})] + 3 [\tan^2(\sqrt{x}) \cdot \sec^2(\sqrt{x}) \cdot \frac{1}{2\sqrt{x}}] (\sqrt{x})$$

$$y' = \frac{\tan^3(\sqrt{x})}{2\sqrt{x}} + \frac{3}{2} [\tan^2(\sqrt{x}) \sec^2(\sqrt{x})]$$

### Exercise Set (3.1)

Using Implicit Function الطال الضمنية

$$1-x+xy-2x^3=2$$

بشتق بالنسبة لـ x

$$1+y+x \frac{dy}{dx} - 6x^2 = 0$$

$$x \frac{dy}{dx} = 6x^2 - y - 1$$

$$\frac{dy}{dx} = \frac{6x^2 - y - 1}{x} = 6x - \frac{y}{x} - \frac{1}{x}$$

$$2-x^3+y^3=3xy^2$$

$$3x^2 + 3y^2 y' = 3y^2 + 3x(2y)y'$$

$$3y^2 y' - 6xy y' = 3y^2 - 3x^2$$

$$y' [3y^2 - 6xy] = 3y^2 - 3x^2$$

$$y' = \frac{3y^2 - 3x^2}{3y^2 - 6xy} \neq$$

$$3-x^2y+3xy^3-x=3$$

$$2xy + x^2 y' + 3y^3 + 3x(3y^2)y' - 1 = 0$$

$$x^2 y' + 9xy^2 y' = -2xy - 3y^3 + 1$$

$$y' [x^2 + 9xy^2] = -2xy - 3y^3 + 1$$

$$y' = \frac{-2xy - 3y^3 + 1}{x^2 + 9xy^2} \neq$$

$$4-\cos(xy^2)=y$$

$$(y^2 + 2xy y') \sin(xy^2) = y'$$

$$-y^2 \sin(xy^2) - 2xy y' \sin(xy^2) = y'$$

$$-y^2 \sin(xy^2) = y' + 2xy y' \sin(xy^2)$$

$$-y^2 \sin(xy^2) = y' [1 + 2xy \sin(xy^2)]$$

$$y' = \frac{-y^2 \sin(xy^2)}{1 + 2xy \sin(xy^2)} \neq$$

$$5-\tan^3(xy^2+y)=x$$

$$3 \tan^2(xy^2+y) \cdot [y^2 + 2xy y' + y'] \cdot \sec^2(xy^2+y) = 1$$

$$\sec^2(xy^2+y) = 1$$

$$3 \tan^2(xy^2+y) \sec^2(xy^2+y) \cdot [y^2 + 2xy y' + y'] =$$

$$3y^2 \tan^2(xy^2+y) \sec^2(xy^2+y) +$$

$$6xy y' \tan^2(xy^2+y) \sec^2(xy^2+y) + 3y' \tan^2(xy^2+y) \cdot \sec^2(xy^2+y) = 1$$

$$6xy y' \tan^2(xy^2+y) \sec^2(xy^2+y) + 3y' \tan^2(xy^2+y) \sec^2(xy^2+y) = 1 - 3y^2 \tan^2(xy^2+y) \sec^2(xy^2+y)$$

$$y' [6xy \tan^2(xy^2+y) \sec^2(xy^2+y) + 3 \tan^2(xy^2+y)]$$

$$= 1 - 3y^2 \tan^2(xy^2+y) \sec^2(xy^2+y)$$

$$3) y = (x^3 - 5)(2x + 3)$$

عنى لمرتبتين

(القاعدة)

$$\frac{d}{dx} [g(x) \cdot f(x)] =$$

$$g'(x) f(x) + f'(x) g(x)$$

مشتقة الناتج مضروبين =

مشتقة الدالة الأولى  $\times$  الدالة الثانية +

مشتقة الدالة الثانية  $\times$  الدالة الأولى

$$(1) y' = (3x^2)(2x+3) + (2)(x^3-5)$$

$$y' = (3x^2)(2x+3) + (2x^3-10)$$

$$y'' = 6x(2x+3) + 2(3x^2) + 6x^2$$

$$y'' = 12x^2 + 18x + 6x^2 + 6x^2$$

$$y'' = 24x^2 + 18x \neq$$

هنا ضربت القوسين الأول  
ثم اشتقته ولازم الاثنين  
نطرحوا نفس الناتج

$$(2) y = (x^3 - 5)(2x + 3) = 2x^4 + 3x^3 - 10x - 15$$

$$y' = 8x^3 + 9x^2 - 10$$

$$y'' = 24x^2 + 18x \neq$$

### Exercises Set (2.4)

$$1- f(x) = (x+1)(x^2 - x + 1)$$

(a) Multiplying and then diff  $\rightarrow$

أضرب ثم فاخذ

$$f(x) = (x+1)(x^2 - x + 1)$$

$$f(x) = x^3 - x^2 + x + x^2 - x + 1$$

$$f(x) = x^3 + 1$$

$$f'(x) = 3x^2 \rightarrow \textcircled{1}$$

استخدم قاعدة لافز

(b) using the Product rule  $\rightarrow$

$$f(x) = (x+1)(x^2 - x + 1)$$

$$f'(x) = (1)(x^2 - x + 1) + (2x - 1)(x + 1) \rightarrow \text{بسط بشكل كامل}$$

$$f'(x) = (x^2 - x + 1) + (2x^2 + 2x - x - 1)$$

$$f'(x) = 3x^2 \rightarrow \textcircled{2}$$

(القاعدة)

$$(f \cdot g)' = f' \cdot g + g' \cdot f$$

$$\textcircled{2} = \textcircled{1} \therefore$$

الحوال الجذبية

### Table (4): Exponential Function

$$\Rightarrow \frac{d}{dx} [e^{F(x)}] = e^{F(x)} \cdot F'(x)$$

$$\Rightarrow \frac{d}{dx} [a^{F(x)}] = a^{F(x)} \cdot F'(x) \cdot \ln(a)$$

$a = \text{Constant}$

$$\Rightarrow \ln(e) = 1$$

الحوال الجذبية العكسية

### Table (5): inverse Trigonometric

$$1 - \frac{d}{dx} [\sin^{-1}(u)] = \frac{1}{\sqrt{1-u^2}} \cdot du$$

$$\frac{d}{dx} [\cos^{-1}(u)] = \frac{-1}{\sqrt{1-u^2}} \cdot du$$

$$2 - \frac{d}{dx} [\tan^{-1}(u)] = \frac{1}{1+u^2} \cdot du$$

$$\frac{d}{dx} [\cot^{-1}(u)] = \frac{-1}{1+u^2} \cdot du$$

$$3 - \frac{d}{dx} [\sec^{-1}(u)] = \frac{1}{u\sqrt{u^2-1}} \cdot du$$

$$\frac{d}{dx} [\csc^{-1}(u)] = \frac{-1}{u\sqrt{u^2-1}} \cdot du$$

(b)

أوجد التكاملات الجزئية

$$1- \int \frac{(3x+13)}{x^2+4x+3} dx$$

السطح أقل من المقام  
∴ التكامل فعلي

← أصفاء المقام

$$\frac{3x+13}{(x+1)(x+3)} = \frac{A}{x+1} + \frac{B}{x+3}$$

$$\text{if } \boxed{x=-1} \quad \boxed{x=-3}$$

نضرب في  $(x+1)(x+3)$ 

$$3x+13 = (x+3)A + (x+1)B$$

$$\text{if } \boxed{x=-1} \quad -3+13 = (-1+3)A \rightarrow 10 = 2A \rightarrow \boxed{A=5}$$

$$\text{if } \boxed{x=-3} \quad -9+13 = (-3+1)B \rightarrow 4 = -2B \rightarrow \boxed{B=-2}$$

$$\int \frac{3x+13}{x^2+4x+3} = 5 \int \frac{1}{x+1} dx - 2 \int \frac{1}{x+3} dx$$

$$= \boxed{5 \ln(x+1) - 2 \ln(x+3) + C}$$

$$2- \int \frac{4x+1}{x^3-x^2-2x} dx$$

السطح أقل من المقام  
∴ التكامل فعلي

← أصفاء المقام

$$\int \frac{4x+1}{x(x^2-x-2)} dx = \int \frac{4x+1}{x(x-2)(x+1)} dx$$

$$\text{if } \boxed{x_1=0} \quad \boxed{x_2=2} \quad \boxed{x_3=-1}$$

$$\frac{4x+1}{x(x-2)(x+1)} = \frac{A}{x} + \frac{B}{x-2} + \frac{C}{x+1}$$

نضرب في  $x(x-2)(x+1)$ 

$$4x+1 = (x-2)(x+1)A + x(x+1)B + x(x-2)C$$

$$\text{if } \boxed{x=0} \quad (4 \times 0) + 1 = (0-2)(0+1)A \rightarrow 1 = -2A \rightarrow \boxed{A = -\frac{1}{2}}$$

$$\text{if } \boxed{x=2} \quad (2 \times 4) + 1 = 2(2+1)B \rightarrow 9 = 6B \rightarrow \boxed{B = \frac{3}{2}}$$

$$\text{if } \boxed{x=-1} \quad (4 \times -1) + 1 = -1(-1-2)C \rightarrow -3 = 3C \rightarrow \boxed{C = -1}$$

$$\int \frac{4x+1}{x^3-x^2-2x} dx = -\frac{1}{2} \int \frac{1}{x} dx + \frac{3}{2} \int \frac{1}{x-2} dx - \int \frac{1}{x+1} dx$$

$$= \boxed{-\frac{1}{2} \ln x + \frac{3}{2} \ln(x-2) - \ln(x+1) + C}$$

(7)

$$\int \frac{5x-1}{x^3(2x-1)^2} dx$$

$$\rightarrow x=0$$

$$x = \frac{1}{2}$$

$$\frac{5x-1}{x^3(2x-1)^2} = \frac{Ax+B}{x^2} + \frac{C}{2x-1} + \frac{D}{(2x-1)^2}$$

→

في معادلة  $x^3(2x-1)^2$

$$5x-1 = (2x-1)^2 Ax + B + x^3(2x-1)C + x^2 D$$

(if  $x=0$ )  $-1 = (2 \cdot 0 - 1)^2 A(0) + B \rightarrow -1 = B$

(if  $x = \frac{1}{2}$ )  $\frac{5}{2} - 1 = (\frac{1}{2})^2 D \rightarrow \frac{3}{2} = \frac{1}{4} D \rightarrow D = 6$

$$5x-1 = (4x^2-4x+1)(Ax+B) + 2Cx^3 - Cx^2 + Dx^2$$

$$5x-1 = 4Ax^3 - 4Ax^2 + Ax + 4Bx^2 - 4Bx + B + 2Cx^3 - Cx^2 + Dx^2$$

معامل  
 $x^3$   
 $= 0$

$$0 = 4A + 2C$$

معامل  
 $x^2$   
 $= 0$

$$0 = -4A + 4B - C + D \rightarrow 0 = -4A - 4 - C + 6 \rightarrow -2 = -4A - C$$

معامل  
 $x$   
 $= 5$

$$5 = A - 4B \rightarrow 5 = A + 4 \rightarrow A = 1 \quad C = -2$$

$$\int \frac{5x-1}{x^3(2x-1)^2} = \frac{1}{2} \int \frac{2x+1}{x^2} dx - \int \frac{2}{2x-1} dx + \int \frac{1}{(2x-1)^2} dx$$

$$= \frac{1}{2} \ln(x^2) - \ln(2x-1) + 6(2x-1)^{-1} + C$$

$$= \frac{1}{2} \ln x^2 - \ln(2x-1) + \frac{6}{2x-1} + C$$

$$\int \frac{5x-1}{x^3(2x-1)^2} dx = \left( \frac{1}{2} \ln \frac{x^2}{2x-1} + \frac{6}{2x-1} + C \right)$$

15-1

\* the Basic Integration Formulas:-

$$(1) \int [f(x)]^n f'(x) dx = \frac{[f(x)]^{n+1}}{n+1} \Rightarrow \int x^n dx = \frac{x^{n+1}}{n+1} + C$$

Ex

$$I = \int \frac{x^5 + x - 2}{\sqrt{x}} dx = \int \left[ \frac{x^5}{x^{\frac{1}{2}}} + \frac{x}{x^{\frac{1}{2}}} - \frac{2}{x^{\frac{1}{2}}} \right] dx = \int (x^{\frac{9}{2}} + x^{\frac{1}{2}} - 2x^{-\frac{1}{2}}) dx$$

$$= \left[ \frac{2}{11} x^{\frac{11}{2}} + \frac{2}{3} x^{\frac{3}{2}} - 4x^{\frac{1}{2}} + C \right]$$

$$I = \int (4x-5)^{35} dx = \frac{1}{4} \int 4(4x-5)^{35} dx = \frac{1}{4} \frac{(4x-5)^{36}}{36} + C$$

$$(2) \int \frac{f'(x)}{f(x)} dx = \ln |f(x)| \Rightarrow \int \frac{dx}{x} = \ln(x)$$

Ex  $I = \int \frac{1 + \cos x}{x + \sin x} dx = \ln |x + \sin x| + C$

$$I = \int \frac{\ln x}{x} dx = \int \overset{\text{مشتق}}{\ln x} \cdot \overset{\text{دالة}}{\frac{1}{x}} dx = \left[ \frac{(\ln x)^2}{2} + C \right]$$

$$(3) \int e^{f(x)} f'(x) dx = e^{f(x)} \Rightarrow \int e^{ax} dx = \frac{e^{ax}}{a}$$

Ex  $I = \int e^{4x} dx = \frac{1}{4} \int 4e^{4x} dx = \frac{e^{4x}}{4} + C$

$$(4) \int a^{bx} dx = \frac{a^{bx}}{b \ln(a)}$$

$$\frac{d}{dx} [b^{ax}] = b^{ax} \ln b \cdot a$$

دالة

Ex:-  $\int 4^{5x} dx = \frac{4^{5x}}{5 \ln 4}$

Note

$$* \int c dx = cx + C$$

إلى كامل ماله جزئي

①

$$\boxed{1} \quad I = \int (x+2) \cos x \, dx$$

$$\begin{array}{lcl} u = x+2 & & dv = \cos x \, dx \\ du = dx & \xleftarrow{\int -} & v = \sin x \end{array}$$

$$I = (x+2) \sin x - \int \sin x \, dx$$

$$\boxed{I = (x+2) \sin x + \cos x + C}$$

$$\boxed{2} \quad I = \int x^2 e^{3x} \, dx$$

$$\begin{array}{lcl} u = x^2 & & dv = e^{3x} \, dx \\ du = 2x \, dx & \xleftarrow{\int} & v = \frac{1}{3} e^{3x} \end{array}$$

$$\boxed{\frac{x^2}{3} e^{3x} - \int \frac{2}{3} x e^{3x} \, dx} \rightarrow \text{①}$$

بالتجزئ مرة آخر

$$\frac{2}{3} \int x e^{3x} \, dx \rightarrow \begin{array}{lcl} u = x & & dv = e^{3x} \, dx \\ du = dx & \xleftarrow{\int} & v = \frac{1}{3} e^{3x} \end{array}$$

$$\frac{2}{3} \left[ \frac{x}{3} e^{3x} - \frac{1}{3} \int e^{3x} \, dx \right]$$

$$I = \frac{x^2}{3} e^{3x} - \frac{2}{3} \left[ \frac{x}{3} e^{3x} - \frac{1}{9} e^{3x} \right]$$

$$= \frac{x^2}{3} e^{3x} - \frac{2}{9} x e^{3x} + \frac{2}{27} e^{3x}$$

$$\boxed{e^{3x} \left( \frac{1}{3} x^2 - \frac{2}{9} x + \frac{2}{27} \right) + C}$$

$$\boxed{6} \int \ln x \, dx$$

3

$$u = \ln x \quad dv = dx$$

$$du = \frac{1}{x} dx \quad v = x$$

$$I = x \ln x - \int \frac{1}{x} \cdot x \, dx$$

$$I = x \ln x - \int dx$$

$$I = x \ln x - x \rightarrow x(\ln x - 1) + C$$

$$\boxed{7} \int x^2 e^x \, dx$$

| <u>u</u> | <u>dv</u> |
|----------|-----------|
| $+x^2$   | $e^x$     |
| $-2x$    | $e^x$     |
| $+2$     | $e^x$     |
| $0$      | $e^x$     |

$$\int x^2 e^x \, dx = x^2 e^x - 2x e^x + 2e^x + C$$

$$\boxed{8} \int \sin^{-1} x \, dx$$

$$u = \sin^{-1} x \quad dv = dx$$

$$du = \frac{1}{\sqrt{1-x^2}} dx \quad v = x$$

$$I = x \sin^{-1} x - \int \frac{x}{\sqrt{1-x^2}} dx$$

$$x \sin^{-1} x - \frac{1}{2} \sin^{-1} \frac{x}{a} + C$$

$$2 \sqrt{1-x^2}$$

$$-\cos^{-1} \frac{x}{a}$$

$$[9] \int e^x \cos(x) dx$$

$$\begin{array}{l} u = e^x \\ du = e^x dx \end{array} \quad \begin{array}{l} dv = \cos(x) dx \\ v = \sin(x) \end{array}$$

$$\boxed{I = e^x \sin(x) - \int e^x \sin(x) dx} \rightarrow \textcircled{1}$$

$$\int e^x \sin(x) dx$$

$$\begin{array}{l} u = e^x \\ du = e^x dx \end{array} \quad \begin{array}{l} dv = \sin x dx \\ v = -\cos x \end{array}$$

$$-e^x \cos x + \left( \int e^x \cos x dx \right)$$

$$\int e^x \cos(x) dx = e^x \sin(x) + e^x \cos x - \int e^x \cos x dx$$

$$2 \int e^x \cos(x) dx = e^x \sin(x) + e^x \cos x$$

$$\boxed{\therefore \int e^x \cos(x) dx = \frac{e^x \sin(x)}{2} + \frac{e^x \cos x}{2} + c}$$

$$* \sin ax \cos bx = \frac{1}{2} [\sin(a+b)x + \sin(a-b)x]$$

$$* \sin ax \sin bx = \frac{1}{2} [\cos(a-b)x - \cos(a+b)x]$$

$$* \cos ax \cos bx = \frac{1}{2} [\cos(a+b)x + \cos(a-b)x]$$

$$I_1 = \int \sin 2x \sin 3x dx$$

$$I_1 = \frac{1}{2} \int [\cos(x) - \cos(5x)] dx$$

$$= \frac{1}{2} \left[ \sin(x) - \frac{\sin(5x)}{5} \right]$$

$$= \left[ \frac{1}{2} \sin(x) - \frac{1}{10} \sin(5x) + c \right]$$

$$I_2 = \int \sin 7x \cos 3x dx$$

$$I_2 = \frac{1}{2} \int [\sin(10x) + \sin(4x)] dx$$

$$I_2 = -\frac{1}{2} \frac{\cos(10x)}{10} - \frac{1}{2} \frac{\cos(4x)}{4} + c$$

$$= \left[ -\frac{1}{20} \cos(10x) - \frac{1}{8} \cos(4x) + c \right]$$

$$① \int \frac{x dx}{\sqrt{x^2+1} - \sqrt{x^2-1}}$$

$$② \int \cot x dx$$

$$③ \int \cos x \cos 3x dx$$

$$④ \int \sin 5x \sin 6x dx$$

$$⑤ \int \frac{1}{x^2} e^{\frac{1}{x}} dx$$

تأهیل

H.W

| Function                    | Integration     |
|-----------------------------|-----------------|
| (1) $\int \sin x dx$        | $[-\cos x + c]$ |
| (2) $\int \cos x dx$        | $[\sin x + c]$  |
| (3) $\int \sec^2 x dx$      | $[\tan x + c]$  |
| (4) $\int \csc^2 x dx$      | $[-\cot x + c]$ |
| (5) $\int \sec x \tan x dx$ | $[\sec x + c]$  |
| (6) $\int \csc x \cot x dx$ | $[\csc x + c]$  |

|                                           |                                           |
|-------------------------------------------|-------------------------------------------|
| (1) $\int \frac{dx}{\sqrt{a^2 - x^2}}$    | $[\sin^{-1} \frac{x}{a} + c]$             |
| (2) $\int \frac{-dx}{\sqrt{a^2 - x^2}}$   | $[-\cos^{-1} \frac{x}{a} + c]$            |
| (3) $\int \frac{dx}{a^2 + x^2}$           | $\frac{1}{a} [\tan^{-1} \frac{x}{a} + c]$ |
| (4) $\int \frac{-dx}{a^2 + x^2}$          | $\frac{1}{a} [\cot^{-1} \frac{x}{a} + c]$ |
| (5) $\int \frac{dx}{x \sqrt{x^2 - a^2}}$  | $\frac{1}{a} [\sec^{-1} \frac{x}{a} + c]$ |
| (6) $\int \frac{-dx}{x \sqrt{x^2 - a^2}}$ | $\frac{1}{a} [\csc^{-1} \frac{x}{a} + c]$ |

### Find the Integration

①  $I = \int \frac{1}{\sqrt{x+1} - \sqrt{x}} dx$   $\xrightarrow{\text{رغبت لالہ}} \int \frac{\sqrt{x+1} + \sqrt{x}}{x+1-x} dx = \int (x+1)^{\frac{1}{2}} + (x)^{\frac{1}{2}} dx$   
 $= \frac{2}{3} (x+1)^{\frac{3}{2}} + \frac{2}{3} x^{\frac{3}{2}} = \frac{2}{3} [(x+1)^{\frac{3}{2}} + x^{\frac{3}{2}}] + c$

②  $I = \int \frac{dx}{e^{-x} + 1}$   $\xrightarrow{\text{رغبت لالہ}} \int \frac{e^x}{1+e^x} dx = [\ln(e^x + 1) + c]$

الحالة الثالثة المقام يحتوي على عوامل  
 $g(x) = (ax^2 + bx + c)(x - x_1)(x - x_2)$  مع الدرجة الثانية غير مكررة

$$\int \frac{P(x)}{g(x)} dx = \frac{Ax+B}{\underbrace{ax^2+bx+c}_{\text{لاشتطال}}} + \frac{C}{(x-x_1)} + \frac{D}{(x-x_2)}$$

$$I = \int \frac{2}{(x-1)(x^2+x+4)}$$

مثال

$$\frac{2}{(x-1)(x^2+x+4)} = \frac{A}{(x-1)} + \frac{Bx+D}{x^2+x+4}$$

$$\begin{matrix} x=1 & x^2+x+4 \rightarrow x(x+1)=-4 \\ & x=0 & x=-5 \end{matrix}$$

بعض المعادلات  $(x-1)(x^2+x+4)$

$$2 = (x^2+x+4)A + (x-1)(Bx+D)$$

$$\text{في } x=1 \quad 2 = (1+1+4)A \rightarrow 2 = 6A \rightarrow A = \frac{1}{3}$$

$$\text{في } x=0 \quad 2 = (4)A + (-1)D \rightarrow 2 = 4 \times \frac{1}{3} - D \rightarrow D = 2 - \frac{4}{3} = \frac{2}{3}$$

$$\begin{aligned} \text{في } x=-5 \quad 2 &= (25-5+4)A + (-5-1)(-5B+D) \\ 2 &= 24 \times \frac{1}{3} + 30B + 6 \times \frac{2}{3} \rightarrow 30B = 2 - 8 - 4 = -10 \quad B = -\frac{1}{3} \end{aligned}$$

$$\int \frac{2}{(x-1)(x^2+x+4)} dx = \frac{1}{3} \int \frac{1}{x-1} dx + \int \frac{-\frac{1}{3}x - \frac{2}{3}}{x^2+x+4} dx$$

$$= \frac{1}{3} \int \frac{1}{x-1} dx - \frac{1}{3} \int \frac{x+2}{x^2+x+4} dx \rightarrow (2x+1) = \frac{1}{2} \int \frac{2x+4}{x^2+x+4} dx$$

$$= \frac{1}{3} \int \frac{1}{x-1} dx - \frac{1}{3} \cdot \frac{1}{2} \int \frac{2x+1}{x^2+x+4} dx - \left( \frac{3}{2} \int \frac{1}{x^2+x+4} dx \right)$$

إكمال مربع  
 $x^2+x+4 \rightarrow (x+\frac{1}{2})^2 + x + \frac{1}{4}$

$$= \frac{1}{3} \ln(x-1) - \frac{1}{6} \ln(x^2+x+4)$$

(5)

$$\int \frac{x^2+1}{(x-1)^2(x+1)^2} dx$$

الحالة الثانية: - عوامل باقيا - بالدرجته الأولى مكررة

if  $x=1$   $x=-1$

بعض أطعمة  $(x-1)(x+1)^2$

$$\frac{x^2+1}{(x-1)^2(x+1)^2} = \frac{A}{(x-1)} + \frac{B}{(x-1)^2} + \frac{C}{(x+1)} + \frac{D}{(x+1)^2}$$

$$x^2+1 = (x-1)(x+1)^2 A + (x+1)^2 B + (x-1)^2(x+1) C + (x-1)^2 D$$

if  $x=-1$   $2 = (-1-1)^2 D \rightarrow 2 = 4D \rightarrow D = \frac{1}{2}$

if  $x=1$   $2 = (2)^2 B \rightarrow 2 = 4B \rightarrow B = \frac{1}{2}$

لنعين بقوات  $x^3-2x^2+1$   $D$

$$x^2+1 = (x-1)(x^2+2x+1)A + (x^2+2x+1)B + (x^2-2x+1)C + (x^2-2x+1)D$$

$$= Ax^3 + 2Ax^2 + Ax - Ax^3 - 2Ax - A + Bx^2 + 2Bx + B + Cx^3 - 2Cx^2 + Cx + Cx^3 - 2Cx + C + Dx^2 - 2Dx + D$$

معامل  $x^3$   
= 0

$$0 = A + C \rightarrow A = -C$$

معامل  $x^2$   
= 1

$$1 = 2Ax^2 + Ax^2 + Bx^2 - 2Cx^2 + Cx^2 + Dx^2$$

$$1 = 3Ax^2 + Bx^2 - Cx^2 + Dx^2$$

$$1 = 3C + \frac{1}{2} - C + \frac{1}{2} \rightarrow -4C = 0 \rightarrow C = 0, A = 0$$

معامل  $x$   
 $x=0$

$$0 = A - 2A + 2B + C - 2C - 2D$$

$$0 = -A + 1 + C - 1 \rightarrow A = 0, C = 0$$

$$1 = -A + B + C + D$$

$$1 = -A + \frac{1}{2} + C + \frac{1}{2} \rightarrow A = 0, C = 0$$

$$\int \frac{x^2+1}{(x-1)^2(x+1)^2} dx = \int \frac{0}{x-1} dx + \frac{1}{2} \int \frac{1}{(x-1)^2} dx + \int \frac{0}{x+1} dx + \frac{1}{2} \int \frac{1}{(x+1)^2} dx$$

$$= \frac{1}{2} \cdot \frac{1}{(x-1)} + \frac{1}{2} \cdot \frac{1}{x+1} + C$$

$$= \frac{1}{2} \left( \frac{(x+1) + (x-1)}{(x-1)(x+1)} \right) = \left( \frac{1}{2} \cdot \frac{2x}{(x-1)^2} + C \right)$$

(5)

(4)

إذا كانت عوامل مقام  $g(x)$  من الدرجة الأولى وبجملها أو كل عامكة

الحالة الثانية

$$g(x) = (ax+b)^n = \frac{A}{(ax+b)} + \frac{B}{(ax+b)^2} + \dots + \frac{C}{(ax+b)^n}$$

درجة أولى  
نضع بسط  
أقل مقام  
وهو ثابت

$$I = \int \frac{x^5 + 2x^3 - 1}{x(x-1)^3} dx$$

مثال

$x=0$  |  $x=-1$   
 $x=1$  |  $x=2$

$$\int \frac{x^5 + 2x^3 - 1}{x(x-1)^3} dx = \frac{A}{x} + \frac{B}{(x-1)} + \frac{C}{(x-1)^2} + \frac{D}{(x-1)^3}$$

بجملها  $x(x-1)^3$ 

$$x^5 + 2x^3 - 1 = (x-1)^3 A + x(x-1)^2 B + x(x-1)C + xD$$

if  $x=0 \rightarrow -1 = (x-1)^3 A \rightarrow -1 = -A \rightarrow A=1$

$x=1 \rightarrow 1+2-1 = D \rightarrow D=2$

$x=-1 \rightarrow -1-2-1 = (-1-1)^3 A + (-1-1)^2 B - (-1-1)C - D$

$$-4 = -8(1) - 4B + C - 2$$

$$-4B + C = 6 \rightarrow ①$$

$x=2 \rightarrow 32+16-1 = (2-1)^3 A + 2(2-1)^2 B + 2(2-1)C + 2D$

$$47 = A + 2B + 2C + 2D = 1 + 2B + 2C + 4$$

$$2B + 2C = 42 \rightarrow ②$$

$$8B - 2C = -12$$

$$10B = 30 \rightarrow B=3$$

بجملها ①  $-2C = -12$

(2)

$$\int \frac{1}{x \sqrt{1+x^3}} dx$$

$$\text{let } y = 1+x^3 \rightarrow 2y dy = 3x^2 dx \rightarrow dx = \frac{2y}{3x^2} dy$$

$$\int \frac{1}{x \sqrt{y^2}} \cdot \frac{2y}{3x^2} dy \rightarrow \int \frac{2y}{3x^3 y} dy \quad x^3 = y-1$$

$$\frac{2}{3} \int \frac{1}{y^2-1} dy \xrightarrow[\text{الجزئية}]{\text{مستعمل باكور}} \frac{2}{3} \int \frac{1}{(y+1)(y-1)} dx$$

$$\frac{\frac{2}{3}}{(y+1)(y-1)} = \frac{A}{y+1} + \frac{B}{y-1}$$

$$\boxed{\text{if } y=-1} \quad \boxed{y=1}$$

لضرب المعادلة  $(y+1)(y-1)$

$$\frac{2}{3} = (y-1)A + (y+1)B$$

$$\text{if } y=-1 \quad \frac{2}{3} = (-1-1)A \rightarrow \frac{2}{3} = -2A \rightarrow A = \frac{2}{3} \times \frac{1}{-2} = -\frac{1}{3}$$

$$\text{if } y=1 \quad \frac{2}{3} = (1+1)B \rightarrow \frac{2}{3} = 2B \rightarrow B = \frac{1}{3}$$

$$\int \frac{\frac{2}{3}}{y^2-1} dy = -\frac{1}{3} \int \frac{1}{y+1} + \frac{1}{3} \int \frac{1}{y-1}$$

$$= -\frac{1}{3} \ln(y+1) + \frac{1}{3} \ln(y-1)$$

$$y = \sqrt{1+x^3}$$

$$= -\frac{1}{3} \ln(\sqrt{1+x^3}+1) + \frac{1}{3} \ln(\sqrt{1+x^3}-1)$$

(أجابه إيجابية) إذا كانت عواما لمقام من 1 - 8 - 11 - 14  
 من حالة انكسر إفير الفعلي (البسيط) أكبر من أو يساوي المقام

(3)

كما يتم استخدام القسمة الطويلة انكسر الفير فعلي

$$\int \dots = \frac{\text{الباقى}}{\text{المقسوم عليه}} + \text{النتائج}$$

مثال أوجد  $I = \int \frac{2x^3 + 2x + 1}{x^2 + 1} dx$

(الحل)

عندى هذا البسط أكبر من المقام فبالقالب عندي انكسر غير فعلي فاستخدم القسمة الطويلة

$$\begin{array}{r} 2x \\ x^2 + 1 \overline{) 2x^3 + \dots + 2x + 1} \\ \underline{2x^3 + \dots + 2x} \phantom{+ 1} \\ \phantom{2x^3 + \dots + 2x} 1 \end{array}$$

النتائج أقل  
 من المقسوم عليه  
 القسمة انتهت

$$\int \frac{2x^3 + 2x + 1}{x^2 + 1} dx = \int 2x dx + \int \frac{1}{x^2 + 1} dx$$

$$= x^2 + \tan^{-1} x + C$$

مثال أوجد  $I = \int \frac{3x^4 - 3x^2 - x}{x^2 - 1} dx$

$$\begin{array}{r} 3x^2 \\ x^2 - 1 \overline{) 3x^4 - \dots - 3x^2 - x} \\ \underline{3x^4 - \dots - 3x^2} \phantom{- x} \\ \phantom{3x^4 - \dots - 3x^2} -x \end{array}$$

أقل من المقسوم  
 عليه  
 القسمة انتهت

$$\int \frac{3x^4 - 3x^2 - x}{x^2 - 1} dx = \int 3x^2 dx - \frac{1}{2} \int \frac{2x}{x^2 - 1} dx$$

$$= x^3 - \frac{1}{2} \ln(x^2 - 1) + C$$

$$3- \int \frac{x^3-1}{4x^3-x} dx \quad \begin{array}{l} \text{البسط ليس قابلاً} \\ \text{للكسر غير فعال} \end{array}$$

← باستخدام القسمة الطويلة

$$\begin{array}{r} \frac{1}{4} \\ 4x^3-x \overline{) x^3 + \dots -1} \\ \underline{\ominus x^3} \quad \oplus \frac{1}{4}x \\ \frac{1}{4}x - 1 \end{array} \rightarrow \begin{array}{l} \text{الباقى أقل من} \\ \text{المقسوم عليه} \\ \therefore \text{انتهت القسمة} \end{array}$$

$$\int \frac{x^3-1}{4x^3-x} dx = \int \frac{1}{4} dx + \int \frac{\frac{1}{4}x-1}{4x^3-x} dx \rightarrow \text{الجزء ٢ كسر فعال نحلل بقايا}$$

$$\int \frac{\frac{1}{4}x-1}{x(4x^2-1)} dx = \int \frac{\frac{1}{4}x-1}{x(2x-1)(2x+1)} dx \quad x=0 \quad x=\frac{1}{2} \quad x=-\frac{1}{2}$$

$$\frac{\frac{1}{4}x-1}{x(2x-1)(2x+1)} = \frac{A}{x} + \frac{B}{2x-1} + \frac{C}{2x+1} \rightarrow \text{نضرب المعادلة بـ } x(2x-1)(2x+1)$$

$$\frac{1}{4}x-1 = (2x-1)(2x+1)A + x(2x+1)B + x(2x-1)C$$

$$\text{if } x=0 \rightarrow -1 = (0-1)(0+1)A \rightarrow -1 = -A \rightarrow A=1$$

$$\text{if } x=\frac{1}{2} \rightarrow \frac{1}{8}-1 = \frac{1}{2}(2 \cdot \frac{1}{2}+1)B \rightarrow \frac{-7}{8} = B \rightarrow B = \frac{-7}{8}$$

$$\text{if } x=-\frac{1}{2} \rightarrow -\frac{1}{8}-1 = \frac{-1}{2}(2 \cdot \frac{-1}{2}-1)C \rightarrow \frac{-9}{8} = C$$

$$\int \frac{x^3-1}{4x^3-x} dx = \frac{1}{4} \int dx + \int \frac{1}{x} dx - \frac{7}{8} \int \frac{1}{2x-1} dx - \frac{9}{8} \int \frac{1}{2x+1} dx$$

$$= \left( \frac{1}{4}x + \ln x - \frac{7}{16} \ln(2x-1) - \frac{9}{16} \ln(2x+1) + C \right)$$

$$\boxed{3} \int x \tan^2 x \, dx$$

(2)

$$\begin{array}{lcl} u = x & & dv = \tan^2 x \, dx \\ du = dx & \xleftarrow{\oplus} & \xrightarrow{\ominus} V = \tan x - x \end{array}$$

$$\begin{aligned} \int \tan^2 x \, dx &= \int \sec^2 x - 1 \, dx \\ &= \tan x - x \end{aligned}$$

$$I = x(\tan x - x) - \int \tan x - x \, dx$$

$$I = x \tan x - x^2 - \left[ \int \tan x \, dx - \int x \, dx \right]$$

$$\boxed{x [\tan x - x] - \left[ \ln \sec x - \frac{x^2}{2} \right] + c}$$

$$\boxed{4} \int x \sec x \tan x \, dx$$

$$\begin{array}{lcl} u = x & & dv = \sec x \tan x \, dx \\ du = dx & \xleftarrow{\quad} & v = \sec x \end{array}$$

$$x \sec x - \int \sec x \, dx$$

$$\boxed{x \sec x - \ln [\sec x + \tan x] + c}$$

$$\boxed{5} \int x e^x \, dx$$

$$\begin{array}{lcl} u = x & & dv = e^x \, dx \\ du = dx & \xleftarrow{\quad} & v = e^x \end{array}$$

$$I = x e^x - \int e^x \, dx$$

$$I = x e^x - e^x = e^x [x - 1] \neq$$

# إكمال المربع

لإيجاد التكاملات  $\int \frac{dx}{\sqrt{ax^2+bx+c}}$  أو على  $\sqrt{ax^2+bx+c}$  فنستخدم طريقة إكمال المربع

$$I = \int \frac{dx}{\sqrt{2+3x-2x^2}}$$

مثال (11)

$$\int \frac{dx}{\sqrt{2(1+\frac{3}{2}x-x^2)}} = \frac{1}{\sqrt{2}} \int \frac{1}{\sqrt{1+\frac{3}{2}x-x^2}}$$

$$\begin{aligned} & \frac{3}{2}x - x^2 \\ & (\frac{3}{4} - x)^2 \\ & (\frac{9}{16} - \frac{3}{2}x + x^2) \\ & - (\frac{3}{4} - x)^2 \end{aligned}$$

$$\frac{1}{\sqrt{2}} \int \frac{1}{\sqrt{1+\frac{3}{2}x-x^2+(\frac{3}{4})^2-(\frac{3}{4})^2}} = \frac{1}{\sqrt{2}} \int \frac{1}{\sqrt{\frac{25}{16}-(x-\frac{3}{4})^2}}$$

التعويض

$$dy = dx \quad y = x - \frac{3}{4}$$

$$\frac{1}{\sqrt{2}} \int \frac{1}{\sqrt{\frac{25}{16}-y^2}} = \frac{1}{\sqrt{2}} \sin^{-1} \frac{y}{5/4} = \frac{1}{\sqrt{2}} \sin^{-1} \frac{x-\frac{3}{4}}{5/4} + C$$

$$I = \int \frac{dx}{2x^2-5x+3}$$

$$\begin{aligned} & (x - \frac{5}{4})^2 \\ & = x^2 - \frac{5}{2}x + \frac{25}{16} \end{aligned}$$

$$\frac{1}{2} \int \frac{1}{x^2 - \frac{5}{2}x + \frac{3}{2}} = \frac{1}{2} \int \frac{1}{x^2 - \frac{5}{2}x + \frac{25}{16} + \frac{3}{2} - \frac{25}{16}} = \frac{1}{2} \int \frac{1}{(x - \frac{5}{4})^2 - \frac{1}{16}} dx$$

$$\text{let } y = x - \frac{5}{4} \rightarrow dy = dx$$

$$\frac{1}{2} \int \frac{1}{y^2 - \frac{1}{16}} dy = -\frac{1}{2} \int \frac{1}{\frac{1}{16} - y^2} dy = \frac{1}{2} \tanh^{-1} \frac{y}{1/4} + C$$

$$a = \frac{1}{4}$$

$$\begin{aligned} & y = \frac{1}{2} \tanh^{-1} 4y + C \\ & = -2 \tanh^{-1} 4y + C \end{aligned}$$

Ex (1)  $I = \int \tan^2 x \, dx$

$$\boxed{\tan^2 x = \sec^2 x - 1} \rightarrow \int (\sec^2 x - 1) \, dx = [\tan x - x] + C$$

(2)  $I = \int \tan x \, dx$

$$\int \frac{\sin x}{\cos x} \, dx = -\ln(\cos x) + C = \ln(\cos x)^{-1} + C = \ln(\sec x) + C$$

(3)  $I = \int \sec x \, dx$

$$\int \sec x \otimes \frac{\sec x + \tan x}{\sec x + \tan x} \, dx = \int \frac{\sec^2 x + \sec x \tan x}{\sec x + \tan x} \, dx = \ln(\sec x + \tan x) + C$$

(4)  $I = \int \cos^2 x \, dx$

(5)  $I = \int \sin^2 x \, dx$

$$\boxed{\cos^2 x = \frac{1}{2}(1 + \cos 2x)}$$

$$\boxed{\sin^2 x = \frac{1}{2}(1 - \cos 2x)}$$

$$\cdot \frac{1}{2} \int (1 + \cos 2x) \, dx$$

$$\frac{1}{2} \int (1 - \cos 2x) \, dx$$

$$\frac{1}{2} \left[ x + \frac{\sin 2x}{2} \right] + C$$

$$\frac{1}{2} \left[ x - \frac{\sin 2x}{2} \right] + C$$

$$= \left[ \frac{x}{2} + \frac{\sin 2x}{4} + C \right]$$

$$= \left[ \frac{x}{2} - \frac{\sin 2x}{4} + C \right]$$

(5)  $I = \int \frac{\sin x + \cos x}{\cos x} \, dx$

$$= \int \left( \frac{\sin x}{\cos x} + 1 \right) \, dx = [-\ln(\cos x) + x + C]$$

$$= \ln(\cos x)^{-1} + x + C = \ln(\sec x) + x + C$$

(1)

## اكسور الجزئية

يستخدم الكسور الجزئية لتكامل لدوال نسبية (الكسرية) ولترى على صورة  $\frac{P(x)}{q(x)}$  حيث  
 للـ  $P(x)$  كثيرات الحدود من المتغير  $(x)$  وتسمى الكسور الجزئية بالكسور الفعلية (الحقيقية)  
 إذا كانت درجة البسط أقل من درجة المقام. وتسمى بالكسور غير الفعلية إذا كانت درجة  
 البسط أكبر من درجة المقام أو تساوي درجة المقام وطريقة الكسور الجزئية تتعامل مع  
 تحليل الكسر الفعلي مباشرة. في حالة غير فعلي يتم استخدام القسمة الطويلة

(مثال) أوجد التكامل  $I = \int \frac{5x+2}{3x^2+4x-4} dx$

(الحل)

$$\frac{5x+2}{3x^2+4x-4} = \frac{5x+2}{(x+2)(3x-2)} = \frac{A}{x+2} + \frac{B}{3x-2} \quad \text{اطلع الجال} \quad x = -2 \quad x = \frac{2}{3}$$

ضرب المعادلة  $(x+2)(3x-2)$

$$5x+2 = (3x-2)A + (x+2)B$$

$x = \frac{2}{3}$   $5x \cdot \frac{2}{3} + 2 = (3x \cdot \frac{2}{3} - 2) + (\frac{2}{3} + 2)B$

$$\frac{10}{3} + 2 = 0 +$$

$$\frac{16}{3} = \frac{8}{3} B \rightarrow B = \frac{16}{3} \times \frac{3}{8} = 2$$

$x = -2$   $-10+2 = (-6-2)A + (-2+2)B$

$$-8 = -8A + 0$$

$$A = 1$$

$$\int \frac{5x+2}{3x^2+4x-4} dx = \int \frac{1}{x+2} dx + \left(\frac{2}{3}\right) \int \frac{3x-2}{3x-2} dx$$

$$= \left[ \ln(x+2) + \frac{2}{3} \ln(3x-2) + C \right]$$

$$\frac{d}{dx}(3x-2) = 3$$

Note

لو البسط أقل من المقام  $\rightarrow \ln$   $\rightarrow$  المقام

$\tan^{-1}()$

المقام يتحلل ويقيم حالة الكسور الجزئية

# Exercise Set (2.3)

1- Find  $\frac{dy}{dx}$

1)  $y = -3x^{12}$

$y' = \frac{dy}{dx} = (-3)(12)x^{12-1}$

$\frac{dy}{dx} = -36x^{11}$

(القاعدة)

①  $\frac{d}{dx}(x^r) = r x^{r-1}$

②  $\frac{d}{dx}(c f(x)) = c f'(x)$   
ثابت

2)  $3x^8 + 2x + 1$

$y' = (3)(8)x^{8-1} + 2x^{1-1} + 0$

$y' = 24x^7 + 2$

(القاعدة)

$\frac{d}{dx}[c] = 0$  منك  
 $\frac{d}{dx}[1] = 0$

3)  $y = \sqrt{x} + \frac{1}{x} = x^{-1/2} + x^{-1}$

$y' = \frac{1}{2\sqrt{x}} - x^{-1-1}$

$y' = \frac{1}{2\sqrt{x}} - x^{-2} = \frac{1}{2\sqrt{x}} - \frac{1}{x^2}$

(القاعدة)

①  $\frac{d}{dx}(\sqrt{x}) = \frac{1}{2\sqrt{x}}$

②  $\frac{d}{dx}(\frac{1}{x}) = -\frac{1}{x^2}$

5)  $y = x^e + \frac{1}{x^{\sqrt{10}}} = x^e + x^{-\sqrt{10}}$

$y' = e x^{e-1} + \sqrt{10} x^{-\sqrt{10}-1}$

$y' = e x^{e-1} - \sqrt{10} x^{-(\sqrt{10}+1)}$

$y' = e x^{e-1} - \frac{\sqrt{10}}{x(\sqrt{10}+1)}$

(القاعدة)

$e \rightarrow$  عبارة عن ثابت

$x^{-n} = \frac{1}{x^n}$

2- Find  $\frac{d^2y}{dx^2}$  المشتقة الثانية

1)  $y = 7x^3 - 5x^2 + x$

$y' = 21x^2 - 10x + 1$

$y'' = 42x - 10 \neq$

2)  $y = \frac{x+1}{x}$

$y = \frac{x}{x} + \frac{1}{x}$

$y = 1 + \frac{1}{x} = x^{-2}$

$y' = 0 - \frac{1}{x^2}$

$y'' = 2x^{-3} = \frac{2}{x^3} \neq$

4)  $y = -3x^{-8} + 2\sqrt{x}$

$y' = (-3)(-8)x^{-8-1} + \frac{2}{2\sqrt{x}}$

$y' = 24x^{-9} + \frac{1}{\sqrt{x}}$

$y' = \frac{24}{x^9} + \frac{1}{\sqrt{x}}$

6)  $y = \frac{x^{3/2} + 2}{x}$  Find  $y'(1)$

$y = \frac{x^{3/2}}{x} + \frac{2}{x}$  للتبسيط بـ  $x$   
الاشتقاق انكسر

$y = x^{3/2-1} + 2x^{-1}$

$y = x^{1/2} + 2x^{-1}$

$y' = \frac{1}{2}x^{1/2-1} - 2x^{-2}$

$y' = \frac{1}{2}x^{-1/2} - \frac{2}{x^2}$

$y'(1) = \frac{1}{2}(1)^{-1/2} - \frac{2}{(1)^2}$

يغير بوضوح  
عن كل  $x$  بواحد (1)  $= \frac{1}{2} - 2 = -\frac{3}{2}$

(2) Find  $F'(x)$

$$(1) f(x) = \left[ \frac{1}{x} + \frac{1}{x^2} \right] (3x^3 + 27)$$

$$\frac{9x^2}{x} + \frac{9x^2}{x^2}$$

$$F(x) = (x^{-1} + x^{-2})(3x^3 + 27)$$

$$= 9x + 9$$

$$F'(x) = (-x^{-2} - 2x^{-3})(3x^3 + 27) + (9x^2)\left(\frac{1}{x} + \frac{1}{x^2}\right)$$

$$F'(x) = -\left(\frac{1}{x^2} + \frac{2}{x^3}\right)(3x^3 + 27) + (9x + 9) \neq$$

$$(2) F(x) = (x-2)(x^2 + 2x + 4)$$

$$F'(x) = (1)(x^2 + 2x + 4) + (2x + 2)(x - 2)$$

$$F'(x) = x^2 + 2x + 4 + 2x^2 - 4x + 2x - 4$$

$$F'(x) = 3x^2$$

$$(3) f(x) = \frac{3x+4}{x^2+1}$$

$$F'(x) = \frac{3(x^2+1) - 2x(3x+4)}{(x^2+1)^2} = \frac{3x^2+3-6x^2-8x}{(x^2+1)^2} = \left[ \frac{-3x^2-8x+3}{(x^2+1)^2} \right]$$

$$(4) f(x) = \frac{2x^2+5}{3x-4}$$

$$F'(x) = \frac{4x(3x-4) - 3(2x^2+5)}{(3x-4)^2} = \frac{12x^2-8x-6x^2-15}{(3x-4)^2} = \left[ \frac{6x^2-8x-15}{(3x-4)^2} \right]$$

$$(5) \text{ Find } \frac{dy}{dx} \Big|_{x=1} \text{ if } y = \frac{2x-1}{x+3}$$

$$y' = \frac{2(x+3) - (2x-1)}{(x+3)^2} = \frac{2x+6-2x-1}{(x+3)^2} = \frac{5}{(x+3)^2}$$

$$y'(1) = \frac{5}{(1+3)^2} = \left[ \frac{5}{16} \right]$$

# Exercises Set (2.5)

Find  $F'(x)$

1-  $F(x) = 4\cos x + 2\sin x$

$F'(x) = -4\sin x + 2\cos x$

3-  $(x^2+1)\sec x$

$F'(x) = 2x(\sec x) + \sec x \tan x (x^2+1)$

$F'(x) = 2x\sec x + \sec x \tan x (x^2+1)$

$F'(x) = \sec x [2x + \tan x (x^2+1)]$

4-  $F(x) = \csc x \cot x$

$F'(x) = -\csc x \cot x (\cot x) - \csc^2 x (\cot x)$

$F'(x) = -\csc x \cot x [\cot x + \csc x]$

5-  $F(x) = \frac{\sec x}{1+\tan x}$

$F'(x) = \frac{\sec x \tan x [1+\tan x] + \sec^2 x [\sec x]}{(1+\tan x)^2}$

$F'(x) = \frac{\sec x \tan x + \sec x \tan^2 x + \sec^3 x}{(1+\tan x)^2}$

$F'(x) = \frac{\sec x [\tan x + \tan^2 x + \sec^2 x]}{(1+\tan x)^2}$

2-  $F(x) = \frac{5-\cos x}{5+\sin x}$

$F'(x) = \frac{\sin x (5+\sin x) - \cos x (5-\cos x)}{(5+\sin x)^2}$

$F'(x) = \frac{5\sin x + \sin^2 x - 5\cos x + \cos^2 x}{(5+\sin x)^2}$

$F'(x) = \frac{5(\sin x - \cos x) + 1}{(5+\sin x)^2}$

$\sin^2 x + \cos^2 x = 1$

# Integration

⇒ iF  $y = 2^{\cos x + \ln x}$   $\frac{dy}{dx}$  using any Method

$$\frac{dy}{dx} = 2^{\cos x + \ln x} \cdot (-\sin x + \frac{1}{x}) \cdot \ln(2) \neq$$

⇒ Find  $\frac{dy}{dx}$  iF

1-  $y = \sin^{-1}(3x)$

$$y' = \frac{1}{\sqrt{1-(3x)^2}} \cdot 3 = \frac{3}{\sqrt{1-(3x)^2}}$$

3-  $y = \csc^{-1}(e^x)$

$$y' = \frac{-1}{e^x \sqrt{(e^x)^2 - 1}}$$

2-  $y = \tan^{-1}(x^3)$

$$y' = \frac{1}{1+(x^3)^2} \cdot 3x^2 = \frac{3x^2}{1+(x^3)^2}$$

تم الانتهاء من جميع مسائل الكتاب  
الكلية [نفا خيل]

Find  $\frac{dy}{dx}$   $\rightarrow$  Implicit diff

$$1- 2x^2 - 3y^2 = 4$$

$$y' \quad 4x - 6yy' = 0 \rightarrow 6yy' = 4x \rightarrow \boxed{y' = \frac{4x}{6y} = \frac{2}{3} \frac{x}{y}}$$

$$3 \quad y'' = \frac{2y - 3xy'}{(3y)^2} = \frac{3 \cdot 2y - (2 \cdot 3x)(\frac{2}{3} \frac{x}{y})}{(3y)^2} = \frac{6y - \frac{4x^2}{y}}{(3y)^2} = \frac{6y^2 - 4x^2}{9y^3} \neq$$

For

$$1- 2 - y + \sin y = x$$

$$y' + \cos y \cdot y' = 1 \rightarrow y' [1 + \cos y] = 1 \rightarrow \boxed{y' = \frac{1}{1 + \cos y}}$$

$$y'' = \frac{0 + \sin y \cdot y'}{(1 + \cos y)^2} = \frac{\sin y \cdot \frac{1}{1 + \cos y}}{(1 + \cos y)^2} = \frac{\sin y}{(1 + \cos y)^3} \neq$$

### Exercise Set (3.2)

\* Find  $\frac{dy}{dx}$  if

$$1- y = \ln \left[ \frac{x}{1+x^2} \right] \quad \text{مشتقة لوغاريتمات كسر فلازم استخدم خواص لوغاريتمات الأول}$$

$$y = \ln(x) - \ln(1+x^2)$$

$$y' = \frac{1}{x} - \frac{2x}{1+x^2} \neq$$

$$2- y = x^2 \log_2(3-2x)$$

$$y' = 2x [\log_2(3-2x)] + x^2 \frac{-2}{\log(2)[3-2x]} = \boxed{2x \log_2(3-2x) - \frac{2x^2}{\log(2)[3-2x]}}$$

$$3-y = \ln(\cos x)$$

$$y' = \frac{-\sin x}{\cos x} = -\tan x$$

$$5-y = \ln(x-1)^3(x^2+1)^4$$

$$y = \ln(x-1)^3 + \ln(x^2+1)^4$$

$$y' = \frac{3(x-1)^2}{(x-1)^3} + \frac{4(x^2+1)^3 \cdot 2x}{(x^2+1)^4}$$

$$y' = \frac{3}{(x-1)} + \frac{8x}{(x^2+1)}$$

$$4-y = \log(\sin^2 x)$$

$$y' = \frac{2\sin x \cos x}{\sin^2 x} = \frac{2\cos x}{\sin x}$$

$$y' = 2\cot x$$

$$6-y = \ln[(\cos^2 x)(\sqrt{1+x^4})]$$

$$y' = \ln(\cos^2 x) + \ln\sqrt{1+x^4}$$

$$y' = \frac{-2\cos x \sin x}{\cos^2 x} + \frac{1}{\sqrt{1+x^4}} \cdot \frac{4x^3}{2\sqrt{1+x^4}}$$

$$y' = \frac{-2\sin x}{\cos x} + \frac{2x^3}{1+x^4}$$

$$y' = -2\tan x + \frac{2x^3}{1+x^4}$$

⇒ Find  $\frac{dy}{dx}$  using logarithmic diff

$$1-y = x\sqrt[3]{1+x^2}$$

→ By using  $\ln$

$$y = \ln x \sqrt[3]{1+x^2} \rightarrow y = \ln x + \ln \sqrt[3]{1+x^2}$$

$$y' = \frac{1}{x} + \frac{\frac{1}{3}(1+x^2)^{-2/3} \cdot 2x}{(1+x^2)^{1/3}} = \frac{1}{x} + \frac{2/3 x}{(1+x^2)} \neq$$

$$(1+x^2)^{1/3}$$

$$2-y = \frac{\sin x \cos x \tan^3 x}{\sqrt{x}}$$

$$y = \ln \left[ \frac{\sin x \cos x \tan^3 x}{\sqrt{x}} \right]$$

$$y = \ln(\sin x) + \ln(\cos x) + \ln(\tan^3 x) - \ln(\sqrt{x})$$

$$y' = \frac{\cos x}{\sin x} - \frac{\sin x}{\cos x} + \frac{3\tan^2 x \sec^2 x}{\tan^3 x} - \frac{1}{\sqrt{x}} \cdot \frac{1}{2\sqrt{x}}$$

$$y' = \cot x - \tan x + \frac{3\sec^2 x}{\tan x} - \frac{1}{2x} \neq$$

القواعد الجبرية للاشتقاق Table (1) :- Rules for diff.

$$1- \frac{d}{dx} [c] = 0$$

$$\frac{d}{dx} (cf) = c f'$$

$$2- (f+g)' = f' + g'$$

$$(f-g)' = f' - g'$$

$$3- (f \cdot g)' = f' \cdot g + g' \cdot f$$

$$\left(\frac{f}{g}\right)' = \frac{f' \cdot g - g' \cdot f}{(g)^2}$$

$$4- \left(\frac{1}{g}\right)' = -\frac{g'}{g^2}$$

$$\frac{d}{dx} [x]^r = r \cdot x^{r-1}$$

الدوال المثلثية Table (2) trigonometric function

$$1- \frac{d}{dx} [\sin(x)] = \cos x$$

$$\frac{d}{dx} [\cos(x)] = -\sin x$$

$$2- \frac{d}{dx} [\tan(x)] = \sec^2 x$$

$$\frac{d}{dx} [\sec(x)] = \sec x \tan x$$

$$3- \frac{d}{dx} [\cot(x)] = -\csc^2 x$$

$$\frac{d}{dx} [\csc(x)] = -\csc x \cot x$$

دوال اللوغاريتمات logarithmic function: - Table (3)

$$1- \log_b (ac) = \log_b a + \log_b c$$

$$2- \log_b \left(\frac{a}{c}\right) = \log_b (a) - \log_b (c)$$

$$3- \log_b (a^r) = r \log_b (a)$$

$$4- \log_b \left(\frac{1}{c}\right) = -\log_b (c)$$

Note

$$1- \log_e x = \ln x = \frac{1}{x}$$

$$2- \log_{10} x = \log x = \frac{1}{x}$$

$$\Rightarrow \frac{d}{dx} [\log (f(x))] = \frac{f'(x)}{f(x)} \quad \begin{matrix} \text{الاشتقاق} \\ \text{الدالة} \end{matrix}$$

$$\Rightarrow \frac{d}{dx} [\log_b (f(x))] = \frac{f'(x)}{\ln b \cdot f(x)}$$

$\log = \ln$